throughout the entire decision process, even though they were read in a particular order. This puts doubt on any straightforward comparison of WOEs to TOEs and SOEs. Therefore, it seems that the term WOE is still the most appropriate.

In cognitive psychology, a widespread theoretical explanation of preference reversals builds on Tversky's (1977) contrast model of similarity (e.g., Houston & Sherman, 1995). This explanation assumes that a choice involves a directional comparison of a *subject* to a *referent*. The comparison is an active search for the subject's features in the referent, where shared features are cancelled and unique features of the subject decide the choice. Because the comparison is assumed to be directional, this means that when the subject has unique positive features, it will be chosen due to the active search for the subject's features in the referent. When the subject has unique negative features, the referent will be chosen due to the salience of the subject's negative features. The evidence for feature matching has mainly come from studies where stimuli were presented as structured feature lists (e.g., Houston & Sherman, 1995), and the stimuli shared positive but not negative features or vice versa. A feature-matching model seems limited to judgments where alternatives can be separated easily into sets of features and inappropriate in, for example, esthetic preference judgment tasks. The SW model is a more general comparison model that does not suffer from dependence on stimulus presentation formats. The present results do not, however, rule out feature matching. A natural direction for future studies is to test these two models against one another.

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TIME- AND SPACE-ORDER EFFECTS IN STIMULUS COMPARISON IN THE LIGHT OF RESPONSE-TIME DATA

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Abstract

As has been found experimentally, magnitude comparison of paired successive or simultaneous stimuli can be described as being based on differential subtractive weighting of the stimulus-induced sensations ψ_1 *and* ψ_2*, causing time- and space-order effects (TOEs and SOEs). In the simplest case, the comparison equation becomes*: $d = s_1 (\psi_1 - \psi_r) - s_2 (\psi_2 - \psi_r)$, *where* ψ_r *is the reference level (ReL). New analyses of response-time data from stimulus comparison experiments (Hellström, 2003) show that* signed response speed*, the inverse of response time with the sign of the subjective difference d, carries similar information on* d *as measures based on transformed proportions of "1st greater," "equal," and "2nd greater" responses. Sensation weighting is similar for fast and slow responses, which suggests that it arises in a preceding processing stage, and strengthens the notion that the weighting, and thereby TOEs and SOEs, is of perceptual origin.*

Hellström (2003) reported five experiments where paired successive or simultaneous stimuli were compared in order to study time- and space-order effects. In these experiments response times (RTs) were recorded but not utilized. Here, the use of RTs in response scaling and analyses of results is discussed. Results from three of the experiments are used for illustration.

Experimental Method

Each participant took part in one session with 1–5 self-paced computerized experiments in irregular order, indicating the longer, louder, etc. stimulus by pressing, with the right hand on a numeric keyboard, "1" (1st or left), "2" (2nd or right), "0" (cannot decide), and then "Enter."

Each experiment had four 24-pair sets, each with its separate ISI or stimulus duration but the same stimulus combinations. All 96 were presented in a pseudorandom order, the same for all participants. Data from three of the experiments are used here.

Experiment 3 (Line length, successive; 39 participants). Stimuli were horizontal orange lines, 2 mm thick, centered on blue background, 100 ms in duration. The first line (Base) had six equal steps within 96 mm ± 60%, and the second line four equal steps within Base ± 22%. ISIs were 100, 300, 900, and 2,700 ms.

Experiment 4 (Tone loudness, successive; 41 participants). Stimuli were 600-Hz tones, 100 ms in duration. The Base tone had four equal sone steps within 11 sones [75 dB(A)] ± 60%, and the second tone six equal steps within Base ± 20%. ISIs were as in Exp. 3.

Experiment 5 (Line length, simultaneous; 31 participants). Stimuli were horizontal orange lines, 2 mm thick, on blue background. A centered orange fixation cross appeared 3 s before the stimuli, lasting 2 s. The lines were centered 96 mm to each side. Their durations were 100, 200, 400, or 800 ms. Equally often the left and the right line was the Base, with three equal steps within 48 mm ±60%. The other line had four equal steps within Base ±13%.

Scaling and Modeling of Subjective Stimulus Differences

To each "1," "2," or "0" response a difference (*d*) value of, in order, +100, -100, or 0 was assigned. The average for a stimulus pair yields *D%,* the percentage difference between "1st greater" and "2nd greater." Hellström (1977) used *D%* to scale *d* in units of the range of a rectangular distribution. As for single pairs *d* should have a more Gaussian-like distribution, and *D%* is bounded by -100 and +100, it will underestimate extreme *d* values.

Response-Distribution Based Measures

Hellström (1977, 1979) described two measures based on Thurstonian assumptions. Denoting the proportions of "first greater" and "second greater" by p_1 and p_2 yields the following consistent estimates [D = mean of d; c = position of category limit; σ_c = SD of d-c; T = mean subjective width of "equal" category; $z_1 = \Phi^{-1}(p_1)$; $z_2 = -\Phi^{-1}(p_2)$]:

$$(D/\sigma_c)^* = (z_1 + z_2)/2; \tag{1}$$

$$(D/T)^* = (z_1 + z_2) / (z_1 - z_2); \tag{2}$$

Hellström (e.g., 1979, 2003) used $(D/T)^*$ and *D%* to fit his *sensation-weighting (SW) model*:

$$d = k\,\{[s_1\,\psi_1 + (1 - s_1)\,\psi_{r1}] - [s_2\,\psi_2 + (1 - s_2)\,\psi_{r2}]\} + b, \tag{3}$$

where k is a scale constant, ψ_1 and ψ_2 the sensation magnitudes of the stimuli, s_1 and s_2 weighting coefficients, and ψ_{r1} and ψ_{r2} the subjective magnitudes that correspond to the current reference levels (*ReL*s). *b* captures a possible judgmental bias. The model parameters can be estimated from linear regression (1) of *d* on the two stimulus magnitudes for each set and (2) of the constant term on the two regression weights across sets (see Hellström, 1979, 2003).

Response-Time Based Measures

Link's wave theory (Link, 1992; cf. Link, 1975, 1978; Link & Heath, 1975) belongs to the family of theories that consider the process behind the two-choice stimulus comparison as a random walk, during which information on the stimulus difference is accumulated, until either of two barriers are reached, at the difference values *A* and –*A*. A distribution-free derivation yields this expression for the *discriminability parameter* ΘA:

$$\Theta A = \ln\,[p_1/(1 - p_1)], \tag{4}$$

which is analogous to the measure obtained by Eq. 1 for the case of no "equal" responses, except that the assumed distribution is logistic rather than Gaussian.

The parameter most closely associated with the stimulus pair is the *mean drift rate*, μ_A. From Link's (1992) analysis of the accumulation process an expression is reached for the expected number of steps, *E(N),* which can be thought of as proportional to the decision time:

$$A = k\,E(N)\,\mu. \tag{5}$$

The barrier is reached at *A* in the proportion of cases p_1, and at –*A* in the proportion $1 - p_1$, so the average position of the barrier reached is $p_1 A + (1-p_1)(-A) = (2p_1 - 1)A$. From this follows

$$\mu_A = (2\,p_1 - 1)\,A / E(N). \tag{6}$$

Thus, μ_A is estimated by $(2p_1 - 1)$ / (mean RT), which will here be called the *L measure*.

Signed response speed. In Link's (1992, ch. 10) analysis of RTs for a given stimulus pair, these are conditional only on the mean drift rate μ_A which is seen as determined by the physical stimulus difference. An alternative approach, which is pursued here, is to focus on the momentary μ values, considering μ as being closely related to *d*. A given stimulus pair gives rise to a *distribution* of values of *d*, and also of μ. Each μ value yields many possible drift paths, and may end up with a response of 1G as well as 2G, but most often with the response compatible with the sign of μ and *d*. Thus, the distributions of μ values conditional on the response–"1st greater" (1G) or "2nd greater" (2G)–will be different.

Using Eq. 5 we may estimate μ from single responses:

$$\mu^* = A / RT \tag{7}$$

for a 1G response ($d>0$), and $\mu^* = -A/RT$ for a 2G response ($d<0$). The measure *100/RT* (for 1G), *-100/RT* (for 2G), and 0 (for "equal") will here be called *signed response speed* (*SRS*).

In using *SRS*, the nondecision part of the RT will here be ignored. Likewise, we let a possible bias (e.g., barrier asymmetry) affect *SRS*. One advantage of *SRS* is that it assigns to each response a graded scale value for μ. Modified measures may be devised, the *L measure* being one. The idea of using RT for scaling is old (see Woodworth & Schlosberg, 1954).

Wave Theory generalized. Estimating μ by using *SRS* or the *L measure* does not require Link's (1992) assumption that stimulus comparison is based on simple subtraction. It is clear from Link's derivations, assuming Poisson waves, that his results hold also when the stimulus comparison is based on a linear combination of waves, for instance a difference between weighted compounds as in the SW model (Eq. 3).

New Analyses of Comparison Data

Goodness of fit for individual data. Linear regressions of *D%* and *SRS* for the data from Exps. 3-5 were computed for each participant and set. Means (*SD*s) over sets of R^2 were: Exp. 3: *SRS* .555 (.096), *D%* .601 (.132), $t(38) = 1.40$, p = .17; Exp. 4: *SRS* .419 (.113), *D%* .350 (.091), $t(40) = 8.30$, $p < .001$; Exp. 5: *SRS* .516 (.069), *D%* .571 (.069), $t(29) = 8.82$, $p < .001$ (paired *t*-tests on *z*-transformed *R* values). Thus, *SRS* generally yielded better fits than *D%*.

Goodness of fit for group data. For each of Exps. 3-5, the response measures were computed on the group data for each pair (the two first-presented pairs, which tended to have very long RTs, there being no practice pairs, were excluded, like all responses faster than 300 ms). Results of multiple regressions are shown in Table 1 in terms of mean adjusted R^2.

Nonlinear relations between measures. To investigate the nature of the relations between the measures, using mean *SRS* as the independent variable, stepwise ($\alpha = 0.05$) polynomial 3rd-degree regressions were fitted to the other five measures over the 94 data points. The β values of the significant nonlinear terms (Q = quadratic, C = cubic) are given in Table 2. With mean *SRS* on the abscissa, $C < 0$ (> 0) means that the plot of the other measure tends to be S-shaped (Z-shaped), and $Q < 0$ (> 0) that it is concave (convex) as seen from below. These shapes reflect the facts that *D%* is bounded by ±100, whereas $(D/\sigma_c)^*$ and $(D/T)^*$ tend to have exaggerated values for large differences (cf. Hellström, 1993). The logit (which disre-

Table 1. Goodness of fit (mean adjusted R^2) for different response measures.

Measure	Exp. 3 (succ. lines)	Exp. 4 (succ. tones)	Exp. 5 (sim. lines)	Mean
Mean *D%*	.807	.787	.816	.803
Mean *SRS*	.802	.805	.834	.814
L measure	.797	.790	.797	.795
Logit	.806	.774	.786	.789
$(D/\sigma_C)^*$ (logistic)	.824	.779	.826	.810
$(D/T)^*$ (logistic)	.791	.738	.872	.800

gards the "equal" responses) is the measure most linearly related to mean *SRS*. According to Wave Theory, the logit does not estimate μ_A directly, but instead ΘA, which need only be monotonically related to μ_A. The linear relation between *SRS* and the logit suggests that both these measures estimate *d* in units of its *SD*.

Exploring the Comparison Mechanisms

Fast vs. slow responses. SRS estimates the drift rate μ for the particular stimulus pair, and thus the subjective difference *d*. It yields good model fits and is computable for every response. It may then be of interest to fit the SW model to *SRS* for fast and slow responses separately.

The responses for each stimulus pair were divided into fast and slow. Mean *SRS* calculated for each category and regressed on the first and second stimulus magnitude. The weight ratios for Exps. 3-4 are plotted in Fig. 1, showing for fast as well as slow responses the same tendencies as in Hellström (2003)–weight ratios being negatively related to ISI. For Exp. 5, they were consistently >1. However, an analysis of β values suggests that for Exp. 3, ISIs 300 and 900 ms, slow responses yield larger β differences than fast (z = 2.20, $p < .05$).

1st greater vs. *2nd greater responses: Partial SRS.* Considering each response category as reflecting a different distribution of μ values, the responses of *1st greater* (*1G*) and *2nd greater* (*2G*) may be used separately to determine values of *partial SRS*, reflecting the *d* values underlying the responses in a particulat category, and thus also the values of the model parameters on those trials that end up with a response in this category.

To assess the *weight ratios*, multiple regressions were computed with the partial *SRS* values as DVs and stimulus magnitudes as IVs, separately for each set. With 1G data from Exp. 3, significant fits of the SW model were obtained for ISI = 100, 300, and 900 ms (mean adj. R^2 over all sets = .3375) and with 2G data for ISI = 300 and 2,700 ms (mean adj. R^2 = .349). For Exp. 4, with 1G significant fits were obtained for ISI = 100 and 900 ms (mean adj.

Table 2. β values for significant cubic © and quadratic (Q) terms in polynomial regressions of response measures on mean *SRS*.

	Exp. 3 (succ.lines)		Exp. 4 (succ. tones)		Exp. 5 (sim. lines)	
Measure \ Term	C	Q	C	Q	C	Q
Mean *D%*	-0.179	-	-0.289	-0.039	-0.307	-
L measure	-	-0.034	-	-0.033	-0.118	-0.026
Logit	-	-	-	-	-	-
$(D/\sigma_C)^*$ (logistic)	0.206	0.056	-	-	-	-
$(D/T)^*$ (logistic)	0.271	0.160	-0.153	-	-	-

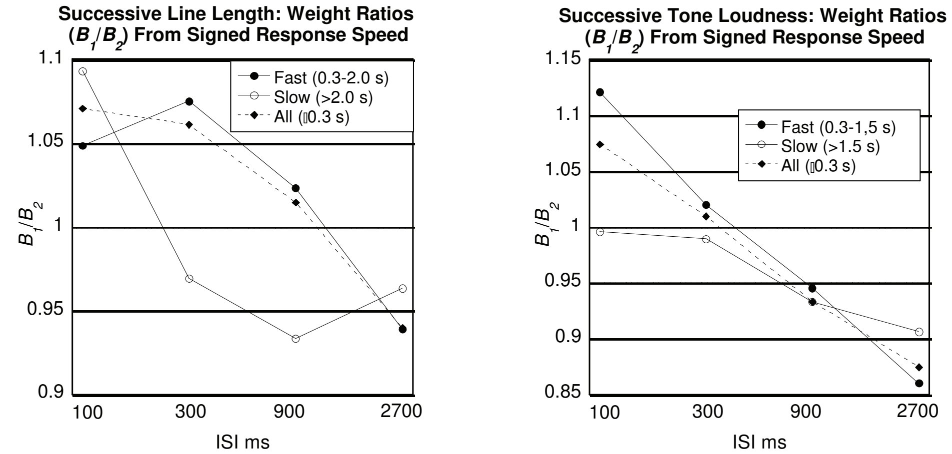

Figure 1. Weight ratios based on SRS for *fast* and *slow* responses.

R^2 = .188) and with 2G for ISI = 300, 900, and 2,700 ms (mean adj. R^2 = . 386). For Exp. 5 no significant fits were obtained. Weight ratios for Exps. 3 and 4 are plotted in Fig. 2.

For Exp. 4, B_1-B_2 tended to be higher for 1G than for 2G (overall: z = 2.92, $p < .003$; 100 ms: z = 1.84, p = .066; 300 ms: z = -0.31, *ns*; 900 ms: z = 2.02, p = .043; 2,700 ms: z = 2.20, p = .028). A repeated-measures ANOVA reveals this tendency as an interaction effect, F (3,75) = 4.179, p = .009. Exp. 3 yields a less clear picture, and no significances.

ReL and b values for 1G and 2G responses. Using the regression technique described in Hellström (2003) on data from Exps. 3 and 4, the *ReL* and *b* values in Eq. 3 were assessed using mean *SRS* for 1G, 2G. and all data. No evidence suggesting $ReL_1 \neq ReL_2$ was found. Estimates of *ReL* and *b* (M±SE_M) are given in Table 3. *b* is near zero for the complete data, but it accounts for most of the difference between 1G and 2G responses, where it seems to represent mean values of the error term, conditional upon the response category. For Exp. 4, a difference in *ReL* between 1G and 2G responses is suggested. Thus, systematic variability in *ReL,* as well as in the weighting pattern, seems to contribute to the variability of *d* between trials, which results in the distribution of responses between the 1G and 2G categories.

Also shown in Table 3 are the corresponding results for fast vs. slow responses. No significant differences in *ReL* or *b* were found for any experiment.

Discussion

Although there are possible differences between weight ratios based on fast and slow responses, these differences are small. The results for successive line length (Fig. 1, left panel) may

Table 3. Parameter values from Eq. 1 (*ReL*; *b*) estimated from signed response speed (*SRS*).

	Exp. 3 (successive lines)		Exp. 4 (successive tones)		Exp. 5 (simult. lines)	
	ReL	*b*	*ReL*	*b*	*ReL*	*b*
1G	133.1±34.5	0.560±0.023	377.2±12.9	0.688±0.015	--	
2G	118.8±15.3	-0.667±0.012	222.7±12.2	-0.619±0.044	--	
Fast	112.9±26.1	-0.113±0.039	252.2±2.8	0.015±0.019	86.9±23.4	0.210±0.087
Slow	126.4±41.5	0.016±0.022	244.9±13.0	0.005±0.012	0.5±1.2	0.027±0.042
All	97.8±9.0	-0.096±0.011	248.6±3.2	0.001±0.018	75.7±23.2	0.143±0.064

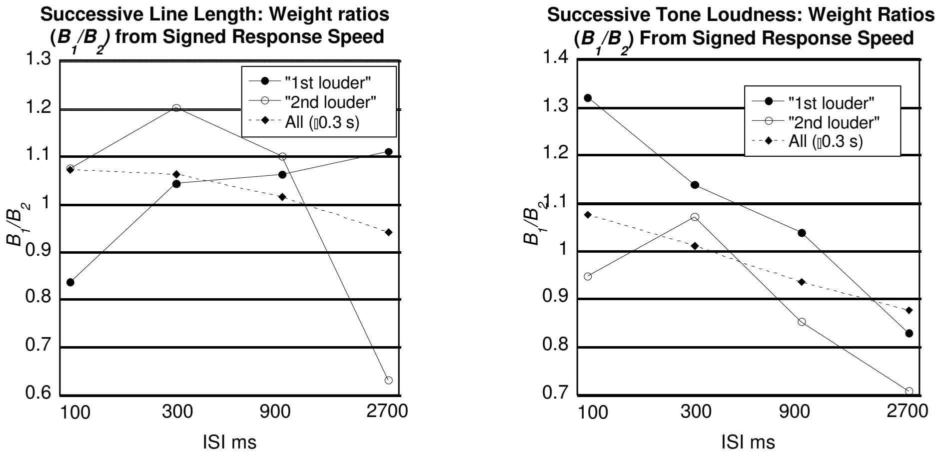

Figure 2. Weight ratios based on partial SRS for *1st greater* and *2nd greater* responses.

suggest an effect of the relative temporal distances from the moment of responding to the 1st and the 2nd stimulus, respectively, which differ the most for medium ISIs. If so, the drift rate might be subject to systematic change in the interval preceding the response.

Still, the dominating impression is that the weighting patterns are robust and stable: for successive lines or tones B_1/B_2 is negatively related to the ISI; for simultaneous lines B_{left}/B_{right} >1. This suggests that the weighting is largely fixed when the response process starts; importantly, this seems to exclude the response process as the origin of the weighting.

The results for Exp. 4 (successive tone loudness) suggest that, at least for this stimulus type, the weighting and reference levels may differ systematically between trials that end up with a response of *1st greater* and *2nd greater*, respectively. This probably reflects a fluctuation of the model parameters between trials.

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QUANTIFICATION OF CATEGORICAL DATA USING DIMENSIONAL ANALYSIS

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Abstract

In previously published works we demonstrated that a modification of Dimensional Analysis (DA) from physics, referred to as Reversed Dimensional Analysis (RDA), can be used to attribute a dimension to a non-physical variable provided measurement data are available, and that such a dimension allows for a meaningful interpretation (cf. Marinov, 2004, 2005). In this article we describe a further step in using DA beyond physics, namely in quantifying categorical data. A theoretical basis and a computational algorithm for such quantification are described. Computer simulations with model data of the kind $X_1=f(X_2,X_3)$, where one of the variables is considered categorical (up to fifteen distinct categories), show that the emerging scale of computed values resembles the structure of the scale of model values, and that the values of the categorical variable can be determined unambiguously.

Dimensional Analysis (DA) and Dimensional Structures (DS)

Consider a set of three variables and dimensional constants describing a phenomenon:

$$f(X_1, X_2, X_3) = 0 \quad \text{or} \quad X_1 = f(X_2, X_3) \tag{1}$$

where, f stands for any functional relation, and each of the three variables can be either dimensionless, or has an independent dimension, or has a dimension dependent on the dimension(s) of one or both of the other two variables. By definition (cf. Krantz et al., 1971), a dimension is independent/dependent when it cannot/can be obtained from the dimensions of the other variables/constants relevant to the phenomenon being investigated. For instance, if velocity (**v**) is defined as distance (**x**) divided by travel time (**t**), i.e. **v=x/t**, then the dimensions of distance (**L**) and time (**T**) can be chosen as independent, while the dimension of velocity (**L/T**) as dependent on the dimensions of distance and time. Similarly, any other couple of dimensions - (**L**) and (**L/T**), or (**T**) and (**L/T**) - can be chosen as independent dimensions, while the remaining one as dependent, which illustrates the relative nature of the notion of independent/dependent dimensions.

According to the Π-theorem of DA (Buckingham, 1914), (1) can be transformed into a relation between a maximum of two dimensionless variables (Π-terms) as follows:

$$F(\Pi_1, \Pi_2) = 0 \quad \text{or} \quad \Pi_1 = F(\Pi_2) \tag{2}$$

where, F stands for any functional relation between Π_1 and Π_2, and the Π 's are dimensionless products of powers of the dimensional variables/constants:

$$\begin{aligned} \Pi_1 &= X_1^{p_{11}} X_2^{p_{12}} X_3^{p_{13}} \\ \Pi_2 &= X_1^{p_{21}} X_2^{p_{22}} X_3^{p_{23}} \end{aligned} \tag{3}$$