C_T contingency depends on the value of the C_C contingency in the stream. When C_C is a good predictor of the outcome ($\Delta P = 1.0$), the observer is likely to indicate that the relationship between the C_T and the outcome is weak. In contrast, when C_C is a poor predictor of the outcome ($\Delta P = 0.0$), the observer is likely to indicate that the relationship between C_T and the outcome is strong. This effect of C_C on the criterion is consistent with variable-criterion accounts in the literature for data generated in other tasks. For example, Treisman (1984) argues that "a criterion is defined not only for a particular judgment, but also for particular conditions under which this judgment may be made. … Thus, the decision criterion may have different values for different sets of circumstances." (pp. 132-133), and he discusses the application of his criterion-setting model to diverse phenomena in the literature.

Acknowledgements

Funding was provided by the National Science and Engineering Council of Canada.

References

Allan, L. G. (1980). A note on measurement of contingency between two binary variables in judgment tasks. *Bulletin of the Psychonomic Society, 15*, 147-149.

Allan, L. G., Hannah, S. D., Crump, M. J. C., & Siegel, S. (2008). The psychophysics of contingency assessment. *Journal of Experimental Psychology: General*, *137*, 226-243.

Allan, L. G, Siegel, S., & Hannah, S. D. (2007). The sad truth about depressive realism. *Quarterly Journal of Experimental Psychology*, *60*, 482-495.

Allan, L. G., & Tangen, J. M. (2005). Judging relationships between events: How do we do it? *Canadian Journal of Experimental Psychology, Special Issue on 2003 Festschrift for Lee R. Brooks, 59,* 22-27.

Alloy, L. B., & Abramson, L. Y. (1979). Judgment of contingency in depressed and nondepressed students: Sadder but wiser? *Journal of Experimental Psychology: General*, *108*, 41-485.

Crump, M. J. C., Hannah, S. D., Allan, L. G., & Hord, L. K. (2007). Contingency judgments on the fly. *Quarterly Journal of Experimental Psychology*. *60*, 753-761.

De Houwer, J., & Beckers, T. (2002). A review of recent developments in research and theories on human contingency learning. *The Quarterly Journal of Experimental Psychology B: Comparative and Physiological Psychology, 55B*, 289-310.

Green, D. M., & Swets, J. A. (1966). *Signal detection theory and psychophysics*. New York, NY: John Wiley and Sons, Inc.

Hannah, S. D., Crump, M. J. C., Allan, L. G., & Siegel, S. (submitted). Cue-interaction effects in contingency judgments using the streamed-trial procedure. *Canadian Journal of Experimental Psychology*.

Tangen, J. M., & Allan, L. G. (2004). Cue interaction and judgments of causality: Contributions of causal and associative processes. *Memory & Cognition, 32,* 107-124.

Treisman, M. (1984). Contingent aftereffects and situationally coded criteria: Discussion paper. In J. Gibbon & L. G. Allan (Eds.), *Timing and Time Perception.* Annals of the New York Academy of Sciences.

Wasserman, E. A. (1990). Attribution of causality to common and distinctive elements of compound stimuli. *Psychological Science*, *1*, 298-302.

REINTERPRETING CORRECT VERSUS ERROR RESPONSE TIMES

Stephen W. Link
University of California, San Diego and McMaster University, Canada
link@ucsd.edu *or* link@mcmaster.ca

Abstract

Early psychological theories of choice and decision followed from developments by Gauss, Fechner, Thurstone, Peterson Birdsall and Fox, and Tanner and Swets. That theoretical structure advanced experimental work in psychophysics and eventually found its way into interpretations of memory performance. A starkly different view of sensory processes rejects this foundation and substitutes for it more recent developments in stochastic processes often viewed as random walks. A critical prediction of the random walk approach concerns the relation between correct and error times. But, these critical predictions are often misunderstood and tests of the predictions misapplied.

In 1821 Karl Gauss published his famous *Theoria combinationis observationum erroribus minimis obnoxiae* (Theory of the Combination of Observations Least Subject to Errors). Gauss's introduction is frankly psychological:

> Certain causes of error are such that their effect on any one observation depends on varying circumstances that seem to have no essential connection with the observation itself. Errors arising in this way are called irregular or random, and they are no more subject to calculation than the circumstances on which they depend. Such errors come from the imperfections of our senses and random external causes, as when shimmering air disturbs our fine vision. (Trans G. W. Stewart)

Nearly 40 years later, in *Elemente der Psychophysik*, pages 104-111, Gustav Fechner developed more extensively Gauss's suggestion that our sensory systems may be perturbed by the same error that affects other measuring devices. This breathtaking application of mathematical ideas to the measurement of mental phenomena defines the origin of scientific investigations of psychological phenomena.

For Gauss the sum of random errors defined the extent of deviations of the observed measure from the true value of the phenomenon to be measured. And, although the individual errors may not be observed, their sum was the cause for variability in repeated measures of the same object. An illustration of Gauss's idea appears in Figure 1. Ten examples of the sum of fifty independent and randomly determined "errors" with mean zero fill the space with stochastic paths illustrating great variability. The end points of each path define the total value of the sum of errors – the possible effect on each of ten individual measurements.

Fechner set himself the task to measure the variabilities that "come from the imperfections of our senses" as postulated by Gauss. His idea launched a thousand experiments and remains today a flagship of experimental psychology. The surprise is that so few know that Fechner invented the idea that launched the thousand ships

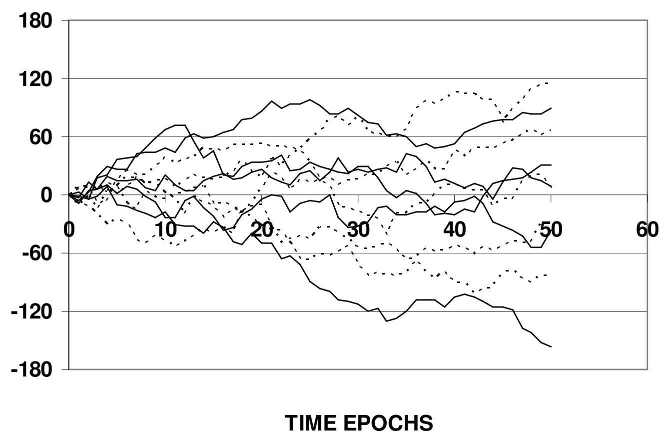

Fig.1. These stochastic paths illustrate the "invisible" sums of errors postulated by Gauss to cause variability in measurements.

Taking Gauss at his word Fechner assumed that the internal manifestation of an external stimulus was perturbed by random error. In order to measure the magnitude of the error he proposed that when the larger of two stimuli is to be chosen the experimental subject chose the stimulus generating the greater internal magnitude. Two stimuli each required internal manifestations and each was subject to the same form of random error. In the face of random error Fechner needed to determine the probability that one stimulus generated an internal value greater than that of the other stimulus. Today we understand that this probability is the volume under the Bivariate Gaussian probability distribution that corresponds to one internal magnitude being greater than another. But in Fechner's day this was a formidable calculation.

The great German mathematician Möbius gave Fechner an insightful solution to determining this rather difficult to calculate probability. Suppose two weights W_A =100g and W_B =110g are compared and have individual internal magnitudes of A and B with average internal magnitudes equal to μ_A and μ_B. The average $(\mu_A+\mu_B)/2 = \mu$ is located midway between μ_A and μ_B. Whenever W_A is judged greater than W_B the subject must believe that the internal magnitude of W_A , the value A, is greater than the value B, the internal value of weight W_B. Mobius proved that the probability that A is greater than B, the probability of judging the lighter weight to actually be the heavier weight, equaled the probability that A was greater than the average $(\mu_A+\mu_B)/2 = \mu$. Making the Newtonian assumption that small differences in the internal magnitudes are a linear function of the external stimulus magnitudes, suggests that the difference between μ and μ_A equals 5g.

Möbius's idea is illustrated in Figure 2. For current researchers this figure will conjure up the idea of Signal Detection Theory á la Green and Swets. But the source of these ideas is the remarkable insight of Fechner and Möbius in the 1850's. Although Fechner's idea involves no actual fixed judgment criterion for the probability of an error, the probability of judging W_A to be greater than W_B can be characterized as if a fixed criterion at.105g acts as a decision criterion against which a single value, is either A or B is compared. The probability of an error remains the same for either A or B.

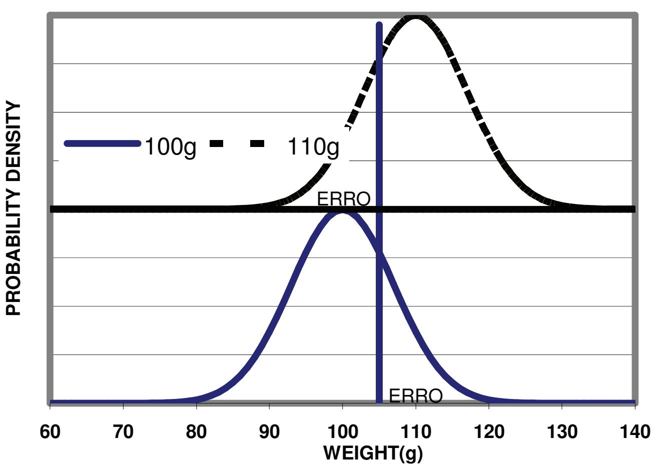

Fig 2. The difficult calculations of internal sensory variability were greatly simplified by Mobius's fixed criterion representation of Fechner's theory.

Fechner's task was to measure variability within the nervous system. His technique was to compute the number of standard deviations 105g deviated from the mean of the 100g weight. For example suppose the proportion of errors equals 0.69. The number of standard deviations corresponding to an error proportion of 0.69 equals 0.50. That is, 0.50 equals the 5g difference divided by the unknown standard deviation σ, $0.50=5g/\sigma$. Therefore $\sigma = 5g/0.69$ and the unknown variability of the sensory system, $\sigma^2 = 52.5g^2$. Psychophysics measured the invisible variability of the nervous system. Psychology became a science.

Later developments extended these ideas. Thurstone sought to apply the idea of pair comparisons to a larger domain of psychological stimulants such as political candidates, foods, crimes, and even weights. But rather than determining the variability in the nervous system, Thurstone determined the psychological difference between stimuli in the standard deviations that were the focus of Fechner's interest. With the advent of Signal Detection Theory (Peterson, Birdsall and Fox, and Tanner and Swets) interest focused on the decision criterion that does not actually exist in the theories of Fechner and Thurstone but proves to be a useful interpretive parameter for results from some psychological experiments. Changes in the position of the criterion, as shown in Figure 2, result in simultaneous changes in the two error proportions.

Ronald Kinchla gets credit for the first test in psychological research of the accumulation of dispersion over time. His random walk model of the variability of the memory of a briefly presented dot of light in a blackened room predicted that the variability of the dispersion increased linearly as a function of time. In a two-choice discrimination experiment Kinchla and Smyzer (1964) proved this prediction to be amazingly accurate.

A Paradigm Shift

A paradigm shift in theorizing about natural phenomena is illustrated in Figure 3. The stochastic paths of Figure 1 are now bounded at maximum and minimum values of 50. Four paths reach the upper threshold of +50, four reach the lower threshold of -50 and two have yet

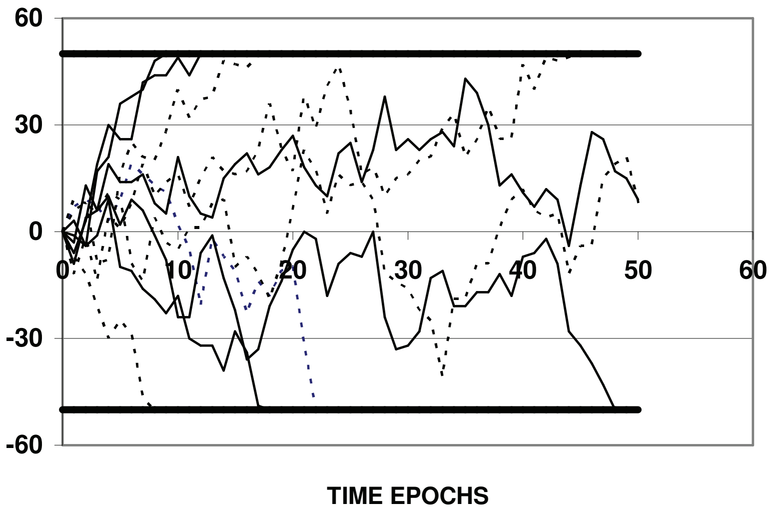

Fig. 3 The Bounded Random Walk with thresholds at -50 and +50.

to reach either threshold by time epoch 50. Only two possible outcomes can occur either +50 or -50. Thus, the Gaussian distribution of random walk end points so well analyzed in the case of Figure 1 suddenly disappears.

In 1900 Louis Bachelier introduced these ideas in a spectacular thesis: “Theorie de la Speculation.” Bachelier derived the variability of stock prices, and determined the probability that the prices would not exceed fixed bounds. Einstein 1905 applied similar concepts to prove that the Gaussian distribution defined Brownian motion - the dispersion of small particles in a viscous medium. In 1915 Schrödinger, and also Smoluchowski, obtained the time taken to first reach a particular amount of dispersion for Brownian motion when a force caused positive drift in the particle. The set of these times is known as the first passage time distribution. In 1923 Weiner established the probabilistic foundation for this continuous time random walk.

The bounded process illustrated in Figure 3 is often characterized as a random walk between absorbing thresholds. For our purposes the characterization in Figure 3 corresponds to opponent processes in the nervous system that captured Fechner’s creative use of differences between sensory signals. But in the new psychophysics, thresholds, magnitudes of capacitance in a cell body, are the bounds on the accumulation of nervous excitation. Once a threshold is exceeded the corresponding cell fires. In application to psychophysical experiments these ideas concerning the neurophysiology of choice are consistent with the theoretical ideas proposed by Rashevsky in the mathematical foundations of biophysics in 1938 and 1960.

In some psychological theories the random walk representation of the choice process did not specify the nature of the dispersion shown in Figure 3. Instead positive or negative increments toward an absorbing state in a Markov process guided predictions (Estes, 1960; Bower, 1960). However difficulties arose when response times, the time taken to reach either of the barriers shown in Figure 3, required integration into the predictions of these theories. Relations between response proportions and response times were both difficult to develop (see for example Audley in 1960) or the predicted relations appeared incompatible with long established experimental data.

The first application of the ideas in Figure 3 to two-choice experiments was Stone’s (1960) bold attempt to apply the ideas of Wald’s Sequential Probability Ratio Test to psychological decision making. Stone’s theory made the same assumptions as did Wald – assumptions such as the experimental subject knowing the probability distributions that defined the stimuli -unlikely to have validity in application to the neurophysiology of the decision process. But, Stone proved that a process such as that envisioned by Wald must predict that correct and error response times for a fixed response must be equal, not only in mean but also in distribution.

For example, imagine the stochastic paths in Figure 3 as resulting from flips of a coin. For coin A the probability of a Head giving a positive increment of size 1 equals p, and the probability of a Tail yielding a negative increment of size -1 equals q = 1-p. A second coin B has probability q of a Head leading to a positive increment and p yielding a negative increment of -1. These two coins may be taken as proxies for two stimuli having equal but opposite effects. Under these conditions the probability that Coin A generates a response by hitting the bound at 50 before hitting the bound at -50 is virtually 1.0 Similarly, the probability the Coin B generates a response by hitting the bound at -50 is virtually 1.0.

However, if Coin B were to hit the bound at 50 the time to hit at 50 would be identical to the time for Coin A to hit at 50, identical in the sense that the distribution of hitting times would be the same for either Coin. Moreover, if the entire process did not begin at zero, as in Figure 3, but shifted to some non-zero position representing a pre-trial bias toward one or the other response the theory still predicts that the distribution of response times conditioned on the response made will be equal for Coins A and B. In terms of the language of Signal Detection Theory, the distribution of response times for Hits and False Alarms must be equal and the distribution of response times for Misses and Correct Rejections must be equal.

In a formidable investigation, Laming (1962) tested a variety of predictions derived from the Wald model under slightly less tight restraints on the distributions giving rise to variability in the stimulus. In Laming’s Experiment 2 two stimuli S_A and S_B (vertical white stripes 4in or 2.83in in height and 0.5in in width) were presented within blocks of 200 trials with different probabilities ranging to 0.75 from 0.25 in steps of 0.125. A total of 4800 trials per condition insured a reasonable number of errors per condition.

Figure 4 illustrates quite clearly that the predictions of equal correct and error response times conditioned on the response made are untrue. But notice that the form of the response time function follows the response made, not the stimulus presented! For example S_A has a correct response time that increases rather linearly with increases in presentation probability. The error response to S_B also increases linearly as a function of the presentation probability for S_A. The two functions appear to differ by nearly a constant. The fact that the error response times always follow the same form as the correct response times suggests that some features of the sequential sampling hypothesis may be correct.

Of course the theoretical problem was to define the nature of the deviation from the predicted equality between correct and error times conditioned on the response made. Link (1975, Link & Heath, 1975) discovered a hidden assumption in the Wald formulation – a hidden assumption also in the Brownian motion developments of Einstein, Schrödinger, and Wiener. This assumption is that the moment generating function of the probability distribution of the increments to the random walk is itself symmetric. Such mgfs as that for the asymmetric binomial walk mentioned above are also symmetric. Thus the “skew” of a probability distribution

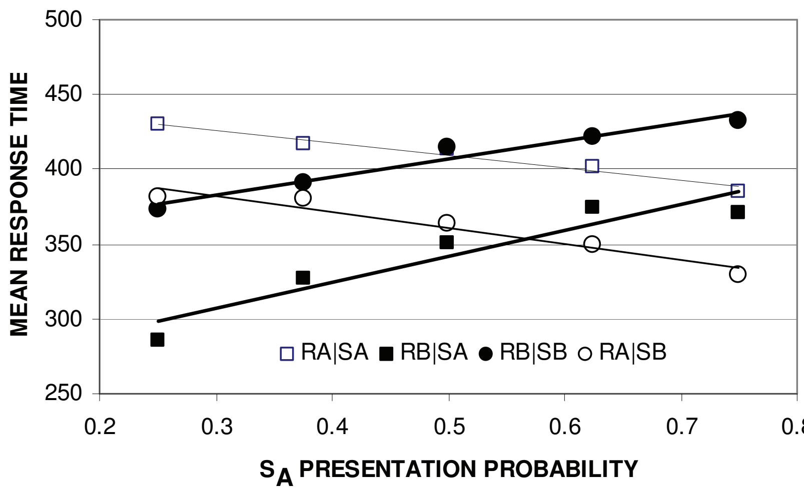

Fig 4. Mean response times follow a similar pattern depending upon the response made.

is not an indicator of whether the mgf is symmetric. When the symmetry of an mgf was relaxed immediate but somewhat difficult to derive theoretical results predicted a difference between correct and error times conditioned on the response. Such differences were to occur in situations where the stimuli being judged were negative images of each other. That is, if stimulus S_A produced increments to the random walk with values X_A then the comparison S_B must generate increments to the random walk $X_B = - X_A$, although the mgf for each must be asymmetric, unlike the coins A and B.

This condition of symmetric stimuli is often misinterpreted and leads to experiments that claim to test the relation between correct and error times but fail to test the proper relation.

References

Bachelier, L. (1900) *Theorie de la speculation*. Ann. Sci. Ec. Norm. Supper., 17(3), 21-86. Paris.

Fechner, G. T. (1860) *Elemente der psychophysik*. Leipzig: Breitkopf & Hartel.

Gauss, K. F. (1821/1995) *Theoria Combinationis Observationum Erroribus Minimis Obnoxiae* (Trans. G. W. Stewart). SIAM.

Kinchla, R. A. & Smyzer, F. A. (1967) A diffusion model of perceptual memory. *Perception and Psychophysics,* 2, 219-229.

Laming, D. R. J. (1968) *Information theory of choice reaction times*. New York: Academic Press.

Link, S. W. (1975) The relative judgment theory of two-choice response time. *Journal of Mathematical Psychology*. 12, 114-135.

Link, S. W. & Heath, R. A. (1975) A sequential theory of psychological discrimination. *Psychometrika*, 40, 77-105.

Stone, M. (1960) Models for choice reaction time. *Psychometrika*, 25, 251-260.

AN ANCIENT PARADOX FOR DISCRIMINATION JUDGEMENTS

Ehtibar N. Dzhafarov
Purdue Universiyt (ehtibar@purdue.edu)

Abstract

According to the pairwise-comparison version of the ancient sorites 'paradox', a stimulus space may contain sequences of stimuli in which any two successive stimuli are not discriminable while the first and the last one are. This hypothesis, often described as an empirical fact, motivates such well-known theoretical constructs as semiorders and interval orders. A rigorous analysis of the notion of (in)discriminability, however, shows that the hypothesis has no empirical justification. Moreover, the pairwise-comparison version of sorites is ruled out by the laws of Regular Minimality (for same-different judgments) and Regular Mediality (for greater-less judgments) proposed by the author as governing principles for comparisons of stimuli belonging to two fixed observation areas (e.g., right-left, or first-second). To deal with all possible forms of perceptual sorites, however, a generalization of Regular Minimality/Mediality is presented which involves multiple observation areas (e.g., multiple spatial locations).

In the 4th century BCE there was a philosopher by the name of Eubulides. All we know of him is that he belonged to Plato's Academia, 'quarrelled' with his contemporary Aristotle, and invented various paradoxes. Most of his paradoxes were mere linguistic trifles, but two of them ('The Liar' and 'The Heap/The Bald') have fascinated scholars ever since. 'The Bald' argument goes as follows: it is obvious that if we remove a single hair from a hairy head we will not make it bald; and conversely, if we add a single hair to a bald head we will not make it hairy; on the other hand this cannot be true because by removing (or adding) hairs one by one we can always transform a hairy head into a bald one (or vice versa). 'The Heap' is a similar argument involving grains of sand forming or not forming a heap. The Greek for 'heap' being *sorites*, philosophers use this word to refer to all such (*soritical*) arguments. In the last 30-40 years there has been an upsurge of interest to sorites (see, e.g., compendiums edited by Beall, 2003, and Keefe & Smith, 1999).

A variant of special interest to psychophysicists is known as *observational sorites* (but I prefer the term '*comparative sorites*'). In its commonly accepted form it states the existence of sequences of stimuli $x_1, x_2, \ldots, x_n$ such that each stimulus is perceptually *matched* by (is *indistinguishable* from) the next one, but x_1 and x_n are perceptually distinct. The reason this may be viewed as a version of Eubulides's ('*classificatory*') sorites is that one can obtain such a *comparative soritical sequence* by starting with the sequence $x_1, x_1, \ldots, x_1$ in which all elements 'obviously' match each other and transforming it step by step, with the changes at every step being 'too small to be noticed':

$$x_1, x_1, x_1 \ldots, x_1 \to x_1, x_2, x_2 \ldots, x_2 \to x_1, x_2, x_3 \ldots, x_3 \to \ldots \to x_1, x_2, x_3 \ldots, x_n.$$

Fechner08v2.indd 42-43 7/11/08 11:51:21 AM