

Fig 4. Mean response times follow a similar pattern depending upon the response made.

is not an indicator of whether the mgf is symmetric. When the symmetry of an mgf was relaxed immediate but somewhat difficult to derive theoretical results predicted a difference between correct and error times conditioned on the response. Such differences were to occur in situations where the stimuli being judged were negative images of each other. That is, if stimulus S_A produced increments to the random walk with values X_A then the comparison S_B must generate increments to the random walk $X_B = -X_A$, although the mgf for each must be asymmetric, unlike the coins A and B.

This condition of symmetric stimuli is often misinterpreted and leads to experiments that claim to test the relation between correct and error times but fail to test the proper relation.

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AN ANCIENT PARADOX FOR DISCRIMINATION JUDGEMENTS

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Abstract

According to the pairwise-comparison version of the ancient sorites 'paradox', a stimulus space may contain sequences of stimuli in which any two successive stimuli are not discriminable while the first and the last one are. This hypothesis, often described as an empirical fact, motivates such well-known theoretical constructs as semiorders and interval orders. A rigorous analysis of the notion of (in)discriminability, however, shows that the hypothesis has no empirical justification. Moreover, the pairwise-comparison version of sorites is ruled out by the laws of Regular Minimality (for same-different judgments) and Regular Mediality (for greater-less judgments) proposed by the author as governing principles for comparisons of stimuli belonging to two fixed observation areas (e.g., right-left, or first-second). To deal with all possible forms of perceptual sorites, however, a generalization of Regular Minimality/Mediality is presented which involves multiple observation areas (e.g., multiple spatial locations).

In the 4th century BCE there was a philosopher by the name of Eubulides. All we know of him is that he belonged to Plato's Academia, 'quarrelled' with his contemporary Aristotle, and invented various paradoxes. Most of his paradoxes were mere linguistic trifles, but two of them ('The Liar' and 'The Heap/The Bald') have fascinated scholars ever since. 'The Bald' argument goes as follows: it is obvious that if we remove a single hair from a hairy head we will not make it bald; and conversely, if we add a single hair to a bald head we will not make it hairy; on the other hand this cannot be true because by removing (or adding) hairs one by one we can always transform a hairy head into a bald one (or vice versa). 'The Heap' is a similar argument involving grains of sand forming or not forming a heap. The Greek for 'heap' being *sorites*, philosophers use this word to refer to all such (*soritical*) arguments. In the last 30-40 years there has been an upsurge of interest to sorites (see, e.g., compendiums edited by Beall, 2003, and Keefe & Smith, 1999).

A variant of special interest to psychophysicists is known as *observational sorites* (but I prefer the term '*comparative sorites*'). In its commonly accepted form it states the existence of sequences of stimuli x_1, x_2, \dots, x_n such that each stimulus is perceptually matched by (is *indistinguishable* from) the next one, but x_1 and x_n are perceptually distinct. The reason this may be viewed as a version of Eubulides's ('*classificatory*') sorites is that one can obtain such a *comparative soritical sequence* by starting with the sequence x_1, x_1, \dots, x_1 in which all elements 'obviously' match each other and transforming it step by step, with the changes at every step being 'too small to be noticed':

$$x_1, x_1, x_1 \dots, x_1 \rightarrow x_1, x_2, x_2 \dots, x_2 \rightarrow x_1, x_2, x_3 \dots, x_3 \rightarrow \dots \rightarrow x_1, x_2, x_3 \dots, x_n.$$

In behavioral sciences the presumed existence of comparative soritical sequences served as a motivation for Luce's celebrated semiorders:

It is certainly well known from psychophysics that if "preference" is taken to mean which of two weights a person believes to be heavier after hefting them, and if "adjacent" weights are properly chosen, say a gram difference in a total weight of many grams, then a subject will be indifferent between any two "adjacent" weights. If indifference were transitive, then he would be unable to detect any weight differences, however great, which is patently false (Luce, 1956).

Most philosophers seem to agree with this: they take the existence of soritical sequences as an indisputable empirical fact and discuss them in terms of intransitivity of the relation 'y matches x' (see, e.g., Goodman, 1951; for 'dissenting' views see Hardin, 1988, and Graph, 2001). The reflexivity and symmetry of 'y matches x' are usually taken for granted.

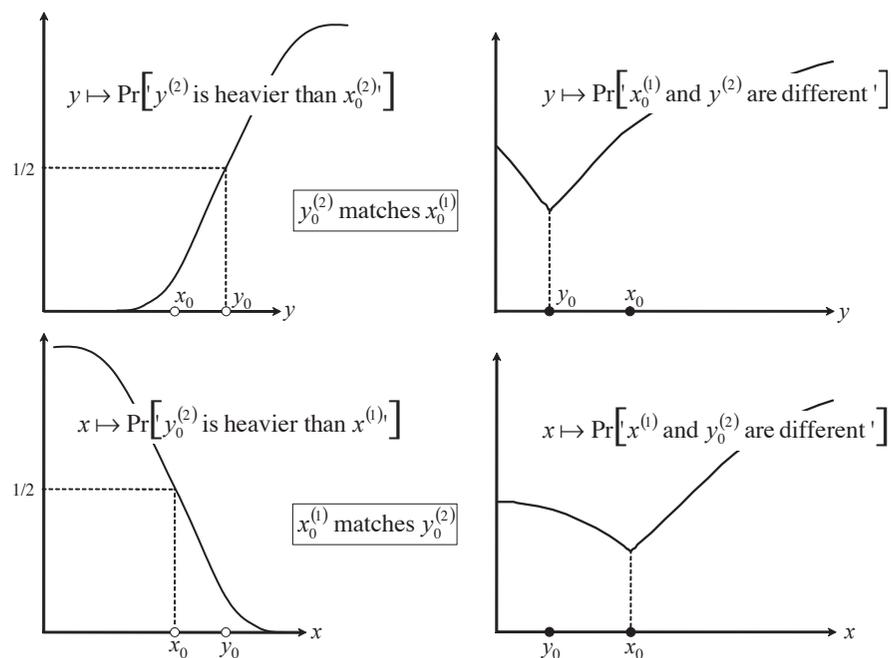


Figure 1: Graphs of psychometric functions $\gamma(x^{(1)}, y^{(2)})$ (left-hand panels) and $\psi(x^{(1)}, y^{(2)})$ (right-hand panels) cross-sectioned at $x = x_0$ (top panels) and $y = y_0$ (bottom panels). For same-different judgments, a point y_0 is a matching point for x_0 if $\psi(x_0^{(1)}, y_0^{(2)})$ is the minimum value for $\psi(x_0^{(1)}, y^{(2)})$ across all stimuli $y^{(2)}$; and a point x_0 is a matching point for y_0 if $\psi(x_0^{(1)}, y_0^{(2)})$ is the minimum value for $\psi(x^{(1)}, y_0^{(2)})$ across all stimuli $x^{(1)}$. Note that, for one and the same pair (x_0, y_0) , y_0 matches x_0 and x_0 matches y_0 . For greater-less judgments, a point y_0 is a matching point for x_0 (and then also x_0 is a matching point for y_0) if $\gamma(x_0^{(1)}, y_0^{(2)}) = \frac{1}{2}$.

There are, however, problems with both the presumed facts and the conceptual apparatus involved in the above quotation from Luce's paper. To avoid common confusions, let us begin with the obvious fact that when a weight x is compared with a weight y the two must belong to different *observation areas* (Dzhafarov, 2002): they either occupy different time intervals (first weight - second weight) or different spatial locations (e.g.,

weight on left palm - weight on right palm). The relation 'y matches x' encompasses in fact two relations: 'y⁽²⁾ matches x⁽¹⁾' and 'y⁽¹⁾ matches x⁽²⁾' (where x and y refer to stimulus values, in this case weights in grams, while the superscripts (1) and (2) encode the observation areas, say left and right). Next, we know that the choice between the judgments 'y⁽²⁾ is heavier than x⁽¹⁾' and 'y⁽²⁾ is not heavier than x⁽¹⁾' in response to a pair of weights $(x^{(1)}, y^{(2)})$ is not determined by $(x^{(1)}, y^{(2)})$ uniquely. Rather, as illustrated in Fig. 1 (left panels) we deal with a *psychometric function*

$$\gamma(x^{(1)}, y^{(2)}) = \Pr[\text{'y}^{(2)} \text{ is heavier than } x^{(1)}],$$

and we define the *matching relation* M (read 'is matched by') as

$$\begin{aligned} x^{(1)}My^{(2)} &\text{ iff } \gamma(x^{(1)}, y^{(2)}) = \frac{1}{2} \\ x^{(2)}My^{(1)} &\text{ iff } \gamma(y^{(1)}, x^{(2)}) = \frac{1}{2} \end{aligned} \quad (1)$$

(The notion of *constant error* reflects the fact that these relations do not imply $x = y$.) The function $y \mapsto \gamma(x^{(1)}, y^{(2)})$ (the argument for $x \mapsto \gamma(x^{(1)}, y^{(2)})$ being analogous) is usually assumed to be strictly increasing in the vicinity of its median, and there are no empirical indications to the contrary. So if $y^{(2)}$ matches $x^{(1)}$ and z differs from y even 'microscopically', then $z^{(2)}$ does not match $x^{(1)}$. The reason many consider comparative sorites 'a fact' is that they overlook a real fact: a 'differential threshold' is not a stimulus subset whose elements all equally match a fixed stimulus in another observation area; rather it is a crude statistical characterization of the rate of increase of a psychometric function near its median (for greater-less comparisons) or minimum (for same-different comparisons).

It follows from (1) that $y^{(2)}$ matches $x^{(1)}$ if and only if $x^{(1)}$ matches $y^{(2)}$, because both are defined by $\gamma(x^{(1)}, y^{(2)}) = \frac{1}{2}$. It follows that a sequence of weights (for definiteness, starting in observation area 1) in which each weight is matched by the next can only be of the forms

$$x^{(1)}, y^{(2)}, x^{(1)}, \dots, y^{(2)}, x^{(1)} \quad \text{or} \quad x^{(1)}, y^{(2)}, x^{(1)}, \dots, x^{(1)}, y^{(2)},$$

where $x^{(1)}My^{(2)}$ and $y^{(2)}Mx^{(1)}$. In the former sequence the first and the last weights are not comparable (as they belong to the same observation area), in the latter sequence the last element does match the first one. Sorites obtains in neither case. Note that if stimuli are identified by both their values and observation areas, then the relation 'y⁽²⁾ matches x⁽¹⁾' is *irreflexive* (y⁽²⁾ can never be the same as x⁽¹⁾), *symmetric* (y⁽²⁾ matches x⁽¹⁾ iff x⁽¹⁾ matches y⁽²⁾), *intransitive* in the common, 'triadic' form (in $x^{(1)}, y^{(2)}, x^{(1)}$ the first and the last elements are not matched because they are not comparable), but *transitive* in the modified, 'tetradic' form $(x^{(1)}, y^{(2)}, x^{(1)}, y^{(2)})$. We see that almost everything in the traditional approach to comparative sorites is incorrect or in need of important qualifications.

This brief analysis implicitly uses the law of *Regular Mediality* for *greater-less comparisons* (Dzhafarov, 2003; Dzhafarov & Colonius, 2006). In the case of *same-different comparisons* (see Fig. 1, right panels), the matching relation M is defined through a psychometric function

$$\psi(x^{(1)}, y^{(2)}) = \Pr[\text{'x}^{(1)} \text{ and } y^{(2)} \text{ are different}']$$

as

$$\begin{aligned} x^{(1)}My^{(2)} &\text{ iff } \psi(x^{(1)}, y^{(2)}) = \min_z \psi(x^{(1)}, z^{(2)}) \\ x^{(2)}My^{(1)} &\text{ iff } \psi(y^{(1)}, x^{(2)}) = \min_z \psi(z^{(1)}, x^{(2)}) \end{aligned} \quad (2)$$

The law of Regular Mediality in this case is replaced with the law of *Regular Minimality* (Dzhafarov, 2002, 2003; Dzhafarov & Colonius, 2005, 2006, 2007), which in essence says that M defined by (2) has precisely the same properties as M defined by (1). This law is less obvious than Regular Mediality (see the exchange between Ennis, 2006, and Dzhafarov, 2006), but there seems to be no evidence contradicting it.

With this, I could end the paper with the conclusion that there is no empirical justification for comparative sorites, and that certain plausible psychophysical principles (Regular Mediality/Minimality) rule it out theoretically. The analysis, however, is not complete, as it is confined to only those pairwise comparisons in which stimuli belong to two fixed observation areas. To handle the issue of comparative sorites we need to generalize Regular Mediality/Minimality to *multiple* (in fact, an *arbitrary set of*) *observation areas*. Consider the example popular with philosophers: a series of colored objects (say, paper strips) arranged side by side along a line, viewed by an observer who may be invited to compare any two of them. In such a paradigm each object is characterized by its color (stimulus value) and by its spatial position along the line (observation area). Certain propositions related to this paradigm explicitly involve more than just two observation areas. Thus, the question of transitivity can be posed in its classical, triadic form: if $x^{(1)}$ is matched by $y^{(2)}$ and $y^{(2)}$ is matched by $z^{(3)}$, would then $z^{(3)}$ match $x^{(1)}$?

The generalization to be presented is taken from a paper I wrote with Damir Dzhafarov (under review as of May 2008). Here I will only present the main definitions and results (in a slightly simplified form), with all proofs and elaborations omitted. The gist of the development is straightforward: a certain class of stimulus spaces is introduced (called *regular well-matched* spaces) and proved to be inconsistent with the idea of soritical sequences.

Let stimuli belong to a set $S \times \Omega$, where S is a set of stimulus values and Ω a set of observation areas. I will continue to use the more convenient $x^{(\omega)}$ in place of (x, ω) for the elements of $S \times \Omega$. The stimulus set $S \times \Omega$ is assumed to be endowed with a binary relation $x^{(\alpha)}My^{(\beta)}$ (read as ‘ x in α is matched by y in β ’). The most basic property of M is

$$x^{(\alpha)}My^{(\beta)} \implies \alpha \neq \beta. \quad (3)$$

The relation $x^{(\alpha)}My^{(\beta)}$ can be defined by the obvious generalizations of (1), (2), or by computations based on other procedures. Thus, $x^{(\alpha)}My^{(\beta)}$ may be defined to hold if and only if $y^{(\beta)}$ is the mean of all adjustments of variable values in the observation area β judged to match a fixed $x^{(\alpha)}$ (see Dzhafarov, 2006).

We begin with two auxiliary notions.

DEFINITION 1 Given a space $(S \times \Omega, M)$, let us call a sequence $x_1^{(\omega_1)}, \dots, x_n^{(\omega_n)}$ *well-matched* if

$$\omega_i \neq \omega_j \implies x_i^{(\omega_i)}Mx_j^{(\omega_j)} \quad (4)$$

for all $i, j \in \{1, \dots, n\}$.

DEFINITION 2 Let us call two stimuli $a^{(\omega)}, b^{(\omega)}$ of a space $(S \times \Omega, M)$ *equivalent*, and write $a^{(\omega)}Eb^{(\omega)}$, if for any $c^{(\iota)} \in S \times \Omega$,

$$c^{(\iota)}Ma^{(\omega)} \iff c^{(\iota)}Mb^{(\omega)}. \quad (5)$$

E is an equivalence relation on $S \times \Omega$. Note that it holds only between stimuli in the same observation area. A classical example of equivalent stimuli we find in metameric

colors. The definition given is weaker than the one adopted in Dzhafarov & Colonius (2005, 2006, 2007), where two equivalent stimuli in an observation area were required to identically compare to all other stimuli (rather than just identically match or not match them).

We use the notions of well-matched sequences and equivalent stimuli to define two classes of stimulus spaces.

DEFINITION 3 A stimulus space $(S \times \Omega, M)$ is *well-matched* if, for any sequence $\alpha, \beta, \gamma \in \Omega$ and any $a^{(\alpha)} \in S \times \Omega$, there is a well-matched sequence $a^{(\alpha)}, b^{(\beta)}, c^{(\gamma)}$.

In particular, in a well-matched space, for any $a^{(\alpha)} \in S \times \Omega$ one can find, in any $\beta \in \Omega$, a $b^{(\beta)} \in S \times \Omega$ such that $a^{(\alpha)}Mb^{(\beta)}$ and $b^{(\beta)}Ma^{(\alpha)}$.

DEFINITION 4 $(S \times \Omega, M)$ is a *regular space* if, for any $a^{(\omega)}, b^{(\omega)}, c^{(\omega')} \in S \times \Omega$ with $\omega \neq \omega'$,

$$a^{(\omega)}Mc^{(\omega')} \wedge b^{(\omega)}Mc^{(\omega')} \implies a^{(\omega)}Eb^{(\omega)}. \quad (6)$$

Well-matchedness and regularity are independent properties. Our primary interest is in the spaces which are both regular and well-matched. The following theorem shows that such spaces have certain ‘nice’ properties.

THEOREM 1 If $(S \times \Omega, M)$ is a regular well-matched space, then

(i) for any $a^{(\omega)}, b^{(\omega')} \in S \times \Omega$,

$$a^{(\omega)}Mb^{(\omega')} \iff b^{(\omega')}Ma^{(\omega)}; \quad (7)$$

(ii) for any $a^{(\omega)}, b^{(\omega)}, c^{(\omega')} \in S \times \Omega$,

$$a^{(\omega)}Mc^{(\omega')} \wedge b^{(\omega)}Mc^{(\omega')} \implies a^{(\omega)}Eb^{(\omega)} \quad (8)$$

and

$$c^{(\omega')}Ma^{(\omega)} \wedge c^{(\omega')}Mb^{(\omega)} \implies a^{(\omega)}Eb^{(\omega)}; \quad (9)$$

(iii) for any $a^{(\omega)}, x^{(\omega)}, b^{(\omega')}, y^{(\omega')} \in S \times \Omega$,

$$a^{(\omega)}Ex^{(\omega)} \wedge b^{(\omega')}Ey^{(\omega')} \implies \{a^{(\omega)}Mb^{(\omega')} \iff x^{(\omega)}My^{(\omega')}\}; \quad (10)$$

in particular,

$$a^{(\omega)}Ex^{(\omega)} \implies \begin{cases} b^{(\omega')}Ma^{(\omega)} \iff b^{(\omega')}Mx^{(\omega)} \\ \text{and} \\ a^{(\omega)}Mb^{(\omega')} \iff x^{(\omega)}Mb^{(\omega')} \end{cases}. \quad (11)$$

We can now give a general definition of a comparative soritical sequence and formulate the main result.

DEFINITION 5 Given a space $(S \times \Omega, M)$, a sequence $x_1^{(\omega_1)}, \dots, x_n^{(\omega_n)}$ with $x_i^{(\omega_i)} \in S \times \Omega$ for $i = 1, \dots, n$, is called *soritical* if

1. $x_i^{(\omega_i)}Mx_{i+1}^{(\omega_{i+1})}$ for $i = 1, \dots, n-1$,
2. $\omega_1 \neq \omega_n$,
3. $\neg x_1^{(\omega_1)}Mx_n^{(\omega_n)}$.

THEOREM 2 One cannot form a soritical sequence in a regular well-matched space.

I make no claim that all empirical pairwise comparison data necessarily have the structure of regular well-matched spaces. My only claim is that there is no empirical evidence to the contrary, because of which the comparative sorites hypothesis is purely speculative. The idea that stimulus spaces are regular and well-matched is also largely speculative, but it has at least been tested for paradigms involving two observation areas. More importantly, it is conceptually much simpler, so it is a better candidate for a default, benchmark hypothesis.

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AGING AND WORKLOAD CAPACITY: DO OLDER ADULTS INTEGRATE VISUAL STIMULI DIFFERENTLY THAN YOUNGER ADULTS?

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Abstract

The effect of aging on response times and on the processing capacity of redundant-visual signals is a neglected theme in the study of aging. In the redundant target design (RTD), an observer detects the presence of a target. A trial can include two (redundant), single, or no-targets. Do older-adults integrate visual stimuli differently than younger-adults? A new approach to capacity (Townsend & Nozawa, 1995) compares the processing of single- and redundant-target trials to compute an index of workload capacity. We discuss the implication of various theories of aging on Townsend's capacity coefficient: Generalized cognitive slowing models with a single age-related slowing equation for all trials; Information degradation models linking sensory loss with cognitive tasks; and Models assuming a decrease in the efficiency of inhibiting distractors. Experimentally, we compare target-detection latencies and Townsend's capacity for younger- and older-adults in RTD, examining the effects of distractor presence and absence for both groups.

Consider the following situation: Mrs. Jones, an elderly pedestrian, is about to cross the road on a dark night. She looks to the left to spot an oncoming vehicle. A single signal (e.g., the front light of the vehicle) is sufficient to stop her from crossing. Two signals (e.g., two lights) constitute redundant superfluous information. However, there is a bulk of evidence to suggest a gain reaped in detection from the presence of multiple targets -- the Redundant Targets Effect (RTE). This speed up may be the result of mere statistical facilitation, or of an interaction in the processing of the two signals. How will the performance of the elderly person compare with that of her 20-year-old grandson, young Mr. Jones? It is reasonable to assume that he will make his decision faster than she will. An age-related slowdown of responses is a common result in many cognitive tasks. But, will the elderly person integrate the visual signals differently than her younger counterpart? In this study, for the first time, we compare the processing of visual redundant information between younger and older adults.

The situation on the road is mimicked in the laboratory via the well-known Redundant Target Design (RTD). Of the set of stimuli, one is defined as the target, and the other as the distractor. On each trial, the observer responds "Yes" when the display contains at least one target; otherwise she or he responds "No." Consequently, a trial in such a design can include two targets (redundant-targets displays), a single-target (single-target displays), or none (no-target displays). Townsend & Nozawa (1995) defined a measure of capacity, $C(t)$, that gauges the extent to which target processing in one channel is impaired [$C(t) < 1$, limited capacity], left unaffected [$C(t) = 1$, unlimited capacity], or improved [$C(t) > 1$, super-capacity] by adding a target in the other channel. Formally, the capacity coefficient $C(t)$, is defined as $C(t) = H_{U,L}(t) / [H_U(t) + H_L(t)]$, $t > 0$, where $H_U(t)$, $H_L(t)$, and $H_{U,L}(t)$ are the integrated hazard functions calculated in the single- and double-target trials, respectively (see Townsend & Nozawa, 1995, for explication). The subscripts U and L refer, respectively, to the upper and lower position of the target. In our study, we compared capacity coefficient values for older and younger adults. Larger regions of super-capacity (larger than unity values of $C(t)$) for older adults will imply that older adults are integrating redundant signals more efficiently than younger adults do. On the other hand, larger regions of severely limited capacity [$C(t) \leq 0.5$] for older adults, will imply that they are not only slower in their responses, but also less