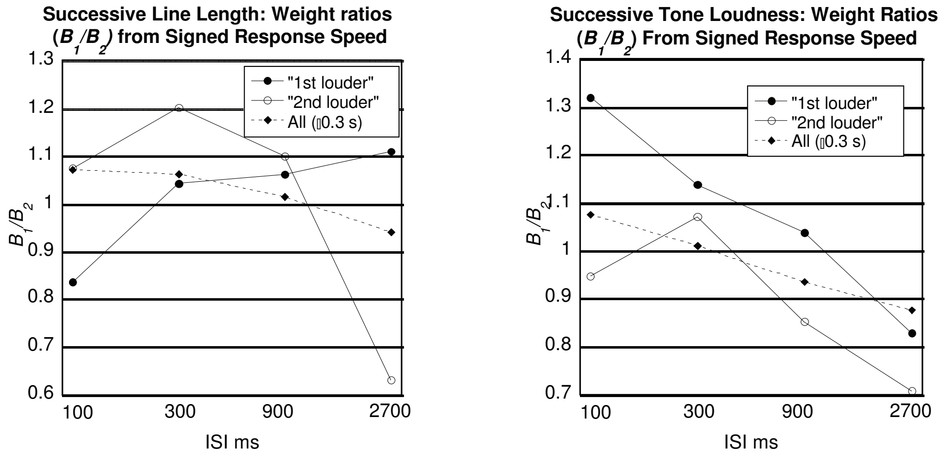

Figure 2. Weight ratios based on partial SRS for *1st greater* and *2nd greater* responses.

suggest an effect of the relative temporal distances from the moment of responding to the 1st and the 2nd stimulus, respectively, which differ the most for medium ISIs. If so, the drift rate might be subject to systematic change in the interval preceding the response.

Still, the dominating impression is that the weighting patterns are robust and stable: for successive lines or tones B_1/B_2 is negatively related to the ISI; for simultaneous lines B_{left}/B_{right} >1. This suggests that the weighting is largely fixed when the response process starts; importantly, this seems to exclude the response process as the origin of the weighting.

The results for Exp. 4 (successive tone loudness) suggest that, at least for this stimulus type, the weighting and reference levels may differ systematically between trials that end up with a response of *1st greater* and *2nd greater*, respectively. This probably reflects a fluctuation of the model parameters between trials.

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QUANTIFICATION OF CATEGORICAL DATA USING DIMENSIONAL ANALYSIS

S. Alex Marinov

Alberta Health & Wellness, Edmonton, Alberta, Canada, marinov@mts.net

Abstract

In previously published works we demonstrated that a modification of Dimensional Analysis (DA) from physics, referred to as Reversed Dimensional Analysis (RDA), can be used to attribute a dimension to a non-physical variable provided measurement data are available, and that such a dimension allows for a meaningful interpretation (cf. Marinov, 2004, 2005). In this article we describe a further step in using DA beyond physics, namely in quantifying categorical data. A theoretical basis and a computational algorithm for such quantification are described. Computer simulations with model data of the kind $X_1=f(X_2,X_3)$*, where one of the variables is considered categorical (up to fifteen distinct categories), show that the emerging scale of computed values resembles the structure of the scale of model values, and that the values of the categorical variable can be determined unambiguously.*

Dimensional Analysis (DA) and Dimensional Structures (DS)

Consider a set of three variables and dimensional constants describing a phenomenon:

$$f(X_1, X_2, X_3) = 0 \quad \text{or} \quad X_1 = f(X_2, X_3) \qquad (1)$$

where, f stands for any functional relation, and each of the three variables can be either dimensionless, or has an independent dimension, or has a dimension dependent on the dimension(s) of one or both of the other two variables. By definition (cf. Krantz et al., 1971), a dimension is independent/dependent when it cannot/can be obtained from the dimensions of the other variables/constants relevant to the phenomenon being investigated. For instance, if velocity (**v**) is defined as distance (**x**) divided by travel time (**t**), i.e. **v=x/t**, then the dimensions of distance (**L**) and time (**T**) can be chosen as independent, while the dimension of velocity (**L/T**) as dependent on the dimensions of distance and time. Similarly, any other couple of dimensions - (**L**) and (**L/T**), or (**T**) and (**L/T**) - can be chosen as independent dimensions, while the remaining one as dependent, which illustrates the relative nature of the notion of independent/dependent dimensions.

According to the Π-theorem of DA (Buckingham, 1914), (1) can be transformed into a relation between a maximum of two dimensionless variables (Π-terms) as follows:

$$F(\Pi_1, \Pi_2) = 0 \quad \text{or} \quad \Pi_1 = F(\Pi_2) \qquad (2)$$

where, F stands for any functional relation between Π_1 and Π_2, and the Π 's are dimensionless products of powers of the dimensional variables/constants:

$$\Pi_1 = X_1^{p_{11}} X_2^{p_{12}} X_3^{p_{13}}$$
$$\Pi_2 = X_1^{p_{21}} X_2^{p_{22}} X_3^{p_{23}} \qquad (3)$$

If all variables/constants in (1), respectively (3), have known dimensions, then the exponents p_{ij} can be found by inspection prior to using measurement data, which is the traditional way of using DA. For instance, if X_1 is a dimensionless variable, X_2 has independent dimension, and the dimension of X_3 depends on the dimension of X_2, then the following relations hold:

$$\begin{array}{lll} \Pi_1 = X_1^1 X_2^0 X_3^0 & \Pi_1 = X_1 & \Pi_1 = X_1 \\ \Pi_2 = X_1^0 X_2^{p_{22}} X_3^{p_{23}} & \Pi_2 = X_2^{p_{22}} X_3^{p_{23}} & \Pi_2 = X_2^{p_{22}} X_3 \end{array} \qquad (4)$$

where Π_2 in the third couple has been redefined by raising both sides to the power of $1/p_{23}$, so that only one exponent appears on the right-hand side. Then, (2) can be rewritten as

$$F(X_1, X_2^{p_{22}} X_3) = 0 \quad \text{or} \quad X_1 = F(X_2^{p_{22}} X_3) \qquad (5)$$

where, the dimensions of X_2 and X_3 are related as $[X_3] = [X_2]^{-p_{22}}$. (Square brackets are conventionally used to denote the dimension of a variable/constant.) Similarly, it can be shown (cf. Marinov 2002) that a maximum of two exponents p_{ij} have values different from **0** or **1** regardless of the combination of dimension types in (1), respectively (3). As well-known, the explicit form of function F cannot be found by DA alone, but only by resorting to data.

As previously shown (Marinov 2002, Table A, see the *Appendix* to this article), because of the restrictions imposed by DA alone, the total number of possible combinations of three variables/constants, each of them having one of the three possible types of dimensions, i.e. **3^3=27**, is reduced to only **12**. Furthermore, because of the relative nature of choosing independent/dependent dimension(s) within certain couplets and triplets of variables, **5** of the combinations in Table A have *dimensional structures* which are *equivalent* to other structures from the list. Namely, **F132** is equivalent to **F123** because either the dimension of X_2 can be chosen as independent (respectively, the dimension of X_3 as dependent), or vice versa. In both cases the dimensional structure of the relationship among X_1, X_2 and X_3 remains essentially the same. Similarly, for the same reason the triplets of independent/dependent dimensions **F223**, **F232** and **F322** can be considered as equivalent. Thus, the general form of a relationship among three variables/constants (1) permits only **7** different dimensional structures.

The term *Dimensional Structure* (DS) is used when all variables/constants have known dimensions. Under the conditions Dimensional Analysis is applied, firstly, such a structure always exists, secondly, it is unambiguously determined by the dimensions of the variables involved, and thirdly, it is established before involving measurement data. In other words, if all variables/constants have known dimensions, the dimensional structure of relation (1) will always be one and only one of the seven different dimensional structures shown in Table A.

Reversed Dimensional Analysis (RDA) and Dimension-Like Structures (DLS) in Data

Now consider relation (1), however, with one of the variables/constants having unknown dimension. In such a case, if (1) possesses any dimensional structure at all, it will be one of the structures listed in Table A. Because of the unknown dimension, the structure cannot be found in the traditional way Dimensional Analysis is applied, i.e. by analyzing the dimensions of the variables alone. If, however, measurement data for all three variables/constants are

available, then the data can be examined for the existence of each of the structures listed in Table A, and a conclusion can be made as to whether a particular structure is present (or absent) in the data set. For instance, (2) can be fit with a function by means of regression analysis, where the set of regression parameters will also include the unknown exponents p_{ij}. Using the Method of Least Squares (cf., Harnett & Murphy, 1993), the following sum has to be minimized

$$\sum (\Pi_1 - F_0(\Pi_2))^2 \qquad (6)$$

where F_0 is an *a priori* chosen fitting function. (The summation indexes are omitted.) Apparently, the above sum is a function of both the usual regression parameters *per se* and the unknown exponents p_{ij}. If it is denoted by Φ, then both the regression parameters ($r_1...r_R$) and the exponents p_{ij} can be found as a set of solutions to the following system of equations:

$$\begin{aligned} &\frac{\partial}{\partial r_i}\Phi(r_1...r_R, p_{ij}) = 0 \;,\; (i = 1...R) \\ &\frac{\partial}{\partial p_{ij}}\Phi(r_1...r_R, p_{ij}) = 0 \;,\; (i = 1,2;\, j = 1,2,3) \end{aligned} \qquad (7)$$

In turn, substituting the numerical values of $r_1...r_R$ and p_{ij} in (2) allows for calculating various statistical characteristics and numerically estimating the goodness of fit by the selected fitting function F_0. As usually practiced in regression analysis, a variety of fitting functions can be tested and the one which provides the best fit can be chosen. If this function satisfies a certain *a priori* chosen goodness of fit criterion, then it can be concluded that the particular dimensional structure is present in the data set, and a dimension can be attributed to the variable of unknown dimension according to a relation of the type of $[X_3] = [X_2]^{-p_{22}}$. Such a structure is regarded as a *Dimension-Like Structure* (DLS), as unlike a *Dimensional Structure* (DS), firstly, it does not necessarily exist; secondly, more than one structure may co-exist simultaneously; and thirdly, the existence of such a structure is established in statistical sense upon applying the above procedure to a particular data set (Marinov, 2002).

An Extension of Reversed Dimensional Analysis (RDA) for Categorical Data

Further in this analysis, consider that one of the variables in (1), say X_3, is a categorical variable. In such a case the method of RDA can be extended to also consider each of the categories as having a unique numeric value different from all others, and respectively consider such values as unknowns in the system of equations (7):

$$\begin{aligned} &\frac{\partial}{\partial r_i}\Phi(r_1...r_R, p_{ij}, C_1...C_S) = 0 \;,\; (i = 1...R) \\ &\frac{\partial}{\partial p_{ij}}\Phi(r_1...r_R, p_{ij}, C_1...C_S) = 0 \;,\; (i = 1,2;\, j = 1,2,3) \\ &\frac{\partial}{\partial C_i}\Phi(r_1...r_R, p_{ij}, C_1...C_S) = 0 \;,\; (i = 1...S) \end{aligned} \qquad (8)$$

Using exactly the same reasoning as described in the previous paragraph, conclusions can be made regarding the existence of a particular DLS in a particular data set, and, if such a structure exists, values attributed to the categories of the categorical variable.

Computer Simulation with Model Data

Model data for $X_1 = f(X_2, X_3)$, where X_3 is considered as a categorical variable, are plotted in Figure 1. The following data generating function was used in modeling $\Pi_1 = F(\Pi_2)$: $F(\Pi_2) = -0.2\Pi_2^2 + 5.0\Pi_2 + 7.0$, where $\Pi_1 = X_1$ and $\Pi_2 = X_2^p X_3$. The simulated 'unknown' exponent was $p = 1.3$, while the 'unknown' values of the categorical variable were $C_1, C_2 ... C_{15}$ =1.8,1.9...3.2. In the denotations of Table A, the data set generated by this function possesses a DLS of type **F123**, i.e. X_1 is dimensionless, X_2 has independent dimension, and X_3 has a dimension dependent on the dimension of X_2.

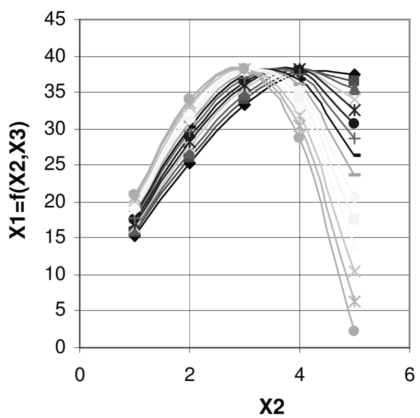

Figure 1. Model data $X_1 = f(X_2, X_3)$

Because of the complex structure of equations (8), instead of looking for an explicit solution for the modeled 'unknowns', a numerical method based on *Genetic Algorithms* (Haupt & Haupt, 1998) was developed to look for the sets of regression parameters, 'unknown' exponents (only one in this case), and 'unknown' values of the categorical variable that minimize the sum in (6) for a chosen fitting function F_0. Two such functions were used: quadratic and power-exponential or *Hoerl's special function* (Daniel & Wood, 1971). In both cases the *Coefficient of Variation for Regression Residuals* was used as a goodness of fit criterion (Harnett & Murphy, 1993). Not unexpectedly, the best fit was produced by the quadratic fitting function. The minimum of the chosen goodness of fit criterion, CV_{min} =0.014, was found at the 3615-th cycle (generation) in a run of 10,000 cycles of the used genetic algorithm. The corresponding set of computed 'unknown' exponent p and 'unknown' values of the categorical variable $C_1, C_2 ... C_{15}$ are shown in Table 1. As evidenced by this table, the value of the computed 'unknown' exponent is practically identical to the model value, while none of the computed 'unknown' values of the categorical variable matches any of the model values. If, however, both series - the model and the computed one - are converted into relative ones, for instance by using the first value in each series as a denominator (for that matter any couple of values in the same position in both series), then the model and computed series become practically identical. In other words, the two scales are identical except for a multiplication by a constant, which is illustrated by the ratio of computed to model values shown in the rightmost column of Table 1. The value of this constant for this particular data set and this particular genetic algorithm run was approximately 5.25. Such a constant seems to be void of meaning specific to a data set but rather indicates a degree of freedom in arbitrarily choosing one (but only one) value as a 'standard'. This is in part supported by the observation that different runs of the genetic algorithm with the same data set may result in different values of the discussed constant.

The finding of fundamental importance in the above analysis is that the structure of the emerging scale of computed values resembles the structure of the scale of model values. Thus, if Reversed Dimensional Analysis is applied to a real data set including a categorical variable of a certain not *a priori* known quantitative structure, if such a structure exists at all, then it can be discovered by this method, and values attributed to the categories of the categorical variable. As the form of Dimensional Analysis used here is applicable to ratio-scaled quantities only, it remains to be studied whether any *apriori* unknown structure can be discovered, or only such that allow ratio-scaling.

Table 1. Comparison of model and computed values for the 'unknown' parameters in $X_1 = f(X_2, X_3)$.

"Unknown" parameter	Model values	Computed values	Ci/C1 model	Ci/C1 computed	Difference (%)	Computed / Model
p	1.300	1.299				
C1	1.800	9.481	1.000	1.000	0.00%	5.27
C2	1.900	9.879	1.056	1.042	1.29%	5.20
C3	2.000	10.500	1.111	1.108	0.32%	5.25
C4	2.100	11.053	1.167	1.166	0.07%	5.26
C5	2.200	11.524	1.222	1.216	0.55%	5.24
C6	2.300	12.079	1.278	1.274	0.29%	5.25
C7	2.400	12.604	1.333	1.329	0.29%	5.25
C8	2.500	13.155	1.389	1.388	0.10%	5.26
C9	2.600	13.647	1.444	1.439	0.35%	5.25
C10	2.700	14.176	1.500	1.495	0.32%	5.25
C11	2.800	14.702	1.556	1.551	0.31%	5.25
C12	2.900	15.232	1.611	1.607	0.28%	5.25
C13	3.000	15.754	1.667	1.662	0.30%	5.25
C14	3.100	16.269	1.722	1.716	0.36%	5.25
C15	3.200	16.798	1.778	1.772	0.34%	5.25

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Appendix

Table A. An exhaustive list of possible combinations of dimensions in a complete set of three variables/constants describing a phenomenon. DS/DLS: Dimensional Structure (DS) when all dimensions are known, or Dimension-Like Structure (DLS) when at least one dimension is unknown. DIMENSION TYPE: **1** – dimensionless variable; **2** – variable or constant with independent dimension, **3** – variable or constant with dependent dimension. (Marinov, 2002)

DS/DLS	DIMENSION TYPE			DIMENSIONLESS EQUATION	EQUIVALENT TO
	X_1	X_2	X_3	$\Pi_1 = F(\Pi_2)$ or $\Pi_1 = C$ *	
F123	**1**	**2**	**3**	$X_1 = F(X_2^{p_{22}} X_3)$	
F132	**1**	**3**	**2**	$X_1 = F(X_2 X_3^{p_{23}})$	**F123**
F213	**2**	**1**	**3**	$X_1^{p_{11}} X_3 = F(X_2)$	
F223	**2**	**2**	**3**	$X_1^{p_{11}} X_2^{p_{12}} X_3 = C$	
F231	**2**	**3**	**1**	$X_1^{p_{11}} X_2 = F(X_3)$	
F232	**2**	**3**	**2**	$X_1^{p_{11}} X_2 X_3^{p_{13}} = C$	**F223**
F233	**2**	**3**	**3**	$X_1^{p_{11}} X_2 = F(X_1^{p_{21}} X_3)$	
F312	**3**	**1**	**2**	$X_1 X_3^{p_{13}} = F(X_2)$	**F213**
F321	**3**	**2**	**1**	$X_1 X_2^{p_{12}} = F(X_3)$	**F231**
F322	**3**	**2**	**2**	$X_1 X_2^{p_{12}} X_3^{p_{13}} = C$	**F223,F232**
F323	**3**	**2**	**3**	$X_1 X_2^{p_{12}} = F(X_2^{p_{22}} X_3)$	
F332	**3**	**3**	**2**	$X_1 X_3^{p_{13}} = F(X_2 X_3^{p_{23}})$	

* C stands for any dimensionless constant.

SUBMICRON-TEXTURE-DISCRIMINATION MECHANISMS IN HUMAN TACTILE PERCEPTION

Tetsu Miyaoka
Shizuoka Institute of Science and Technology
miyaoka@cs.sist.ac.jp

Abstract

The purpose of the present study was to determine the detection thresholds of very-fine textures in human tactile perception and to investigate mechanisms for discrimination. Three experiments were performed. In Experiment 1, fine-abrasive papers with particle sizes between 0.1 and 9 μm were used as stimuli. Six subjects touched the stimuli with their index or middle fingers preferred and judged the roughness with the two-alternative, forced-choice technique. The finger temperature was 35ºC and the detection threshold was 0.84 μm. In Experiment 2 and 3, the experimental procedure was similar to Experiment 1, six subjects judged the roughness of fine-abrasive papers when the finger temperatures were 18ºC and 12ºC, respectively. The detection thresholds obtained in Experiment 2 and 3 were 1.51 and 1.44 μm, respectively. The results of three experiments showed that the mechanoreceptor which detected submicron textures was the Pacinian corpuscle.

Tactile texture perception is divided into two types: coarse-texture perception and fine-texture perception (Hollins et al., 2001; Miyaoka, 1994; Miyaoka et al., 1999). Coarse-texture perception was intensively studied and SA I (slowly-adapting-type-I unit; the Merkel disk) was identified as the mediating receptor (Blake et al., 1997; Connor et al., 1990). Hollins et al. studied fine-texture perception and proposed that FA II (fast-adapting-type-II unit; the Pacinian corpuscle) was the probable receptor (Hollins et al., 2001; Bensmaïa et al., 2005).

The purpose of this study was to determine the detection thresholds of very-fine textures and to investigate mechanisms that make the discrimination of fine-texture possible.

Experiment 1

The purpose of Experiment 1 was to determine the tactile-detection threshold of very-fine textures when the skin temperature was 35ºC.

Method

Subjects: Five males and one female in their twenties participated in Experiment 1.
Stimuli and apparatus: The stimuli were six abrasive papers (Sumitomo 3-M). The grit values assigned by the manufacturer were 2000, 4000, 8000, 10000, 15000, and 20000; representing corresponding average particle sizes of 9, 3, 1, 0.5, 0.3, and 0.1 μm. Observation with a scanning electron microscope (JEOL, JSM-5610LV) confirmed that the particle sizes of each paper corresponded to the particle-size values reported by the manufacturer. The abrasive papers were cut into 20 × 20-mm squares and each piece was attached to an aluminum plate sized 20 × 40-mm. In the experimental trial, two of the six abrasive papers were selected and placed on the experimental device.