R. Duncan Luce
(1925 – 2012)
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IN MEMORIAM

R. Duncan Luce
(1925-2012)

R. Duncan Luce, age 87, awarded the National Medal of Science in 2003, passed away in his sleep at his home in Irvine, California on August 11, 2012. He is survived by his wife Carolyn, daughter Aurora, her husband, and two granddaughters. At a family memorial he was eulogized by his friends and colleagues as a generous, thoughtful, scientist with a wry sense of very good humor and a happy family life.

Duncan Luce is best known in psychology for his formal approach to psychological theorizing. This development seems an unlikely outcome for a young student that entered the Massachusetts Institute of Technology at age 17, graduating three years later with his Bachelor of Science in Aeronautical Engineering. Following a short stint in the US Navy he re-entered MIT in mathematics and produced, in 1950, his PhD thesis entitled *On semigroups*. What seems so remarkable in retrospect is that his first publication in 1949, with A. D. Perry, was "A method of matrix analysis of group structure," which appeared in *Psychometrika*. In 1950 he published "Connectivity and generalized cliques in sociometric group structure" also in *Psychometrika*. Semigroups, group structure, sociometric group structure: Is this the beginning of a career?

His amazing sequence of contributions to a more formal approach to psychological theorizing followed. The contributions may be thought of in several distinct and yet mutually influential groups. During the 1950's his co-development of books and articles about mathematical principles applied to psychology yielded a basis for later works that must be viewed as some of the major contributions to the field in the 20th century. At the same time his contributions to mathematics, economics and measurement took full form. His influential *Games and Decisions* with Raiffa in 1957 set the stage for his wide recognition in Econometrics as did his previous *Econometrika* articles "Semiorders and a theory of utility discrimination" (1956) and "A probabilistic theory of utility" (1958) that preceded by one year his famous "little red book" *Individual Choice Behavior* (1959). These major accomplishments are only a refection of the amazing theoretical work created during the 1950's.

His voluminous work in psychophysics begins in the 1950's in a joint publication with Ward Edwards, "The derivation of subjective scales from just noticeable differences" appearing in *Psychological Review* (1958). In 1959 "A probabilistic theory of utility and its relationship to Fechnerian scaling", and "Response latencies and probabilities" appeared in books and "On the possible psychophysical laws" in *Psychological Review*.

One of the most fundamental of Luce's contributions, and a basis for much of his later thinking, was formalized in *Individual Choice Behavior*. This is the *choice axiom* (henceforth, simply AXIOM). The AXIOM is beguiling in its simplicity. In fact, on first reading of its representation in ICB, a novice would often require a bit of reflection to discern that it was not simply a statement of conditional probability theory. Omitting details such as when a choice object in a set has 0 probability of being chosen, we follow Luce in expressing the AXIOM as

$$P(R|X) = P(R|S) \cdot P(S|X) \text{ where } R \subseteq S \subseteq X.$$

The full AXIOM, despite its innocent appearance, bears the strong consequence that there exists a ratio scaled set of *values* on the objects $x \in X$ such that $v(x)$ is the *preference of* x_i and the

probability of choosing x from any particular subset S of X, including X itself is

$$P(x|S) = v(x_i)/(\sum v(x_j)),$$

where j runs over the index associated with the members of S, including x_i itself.

This formulation is sometimes referred to as the *strong utility model,* where the v's are interpreted as utilities. Moreover again, with provisions for dealing with probability-of-choice of 1 or 0 cases, the AXIOM is also equivalent to the *constant ratio rule,* which is defined by the property that $P(x|S)/P(y|S) = P(x, y)/P(y, x)$, where say, $P(x, y)$ is the probability that x is chosen over y when only the two are presented as alternatives. This strong but elementary appearing rendition of the AXIOM, especially in its strong utility formulation, sometimes misleads the reader into falsely supposing that the AXIOM is only the rule of normalizing a set of numbers so that they add to 1.

It is true that the normalization of positive numbers so that they sum to 1 (which in psychological context, I'll refer to as the "ratio of strengths" principle) was ubiquitous in science and math for quite a spell, to say the least. However, there are two main contributions that render choice theory a highly strategic psychological movement of the twentieth century. The essence of the first branch begins with Roger Shepard's seminal doctoral thesis where what is now referred to as the "similarity choice model" is given birth. The second branch lies within the mathematical structure of ICB.

In some ways it is more straightforward to start with the second branch: The AXIOM is substantially deeper and more consequential than a ratio of strengths quantity by itself. As the above modest descriptions intimates, and the details of ICB demonstrates, it implies a profound type of independence ("invariance" might be a preferable term) among choice objects, even as the set of such objects expands or contracts. The redoubtable constraints imposed on choice and preference behavior by this axiom are responsible for a wide terrain of decision and to some extent foundational measurement research in economics, psychology (especially decision making) and a number of other fields.

When the "strength of preference" value is represented as a product of a similarity value and a response bias value, the "similarity choice model" is created and the model moves from being applicable "only" to valenced choice objects, to sizeable arenas in perception and cognition (e.g., detection, identification, categorization, etc.), as pre-figured by Roger Shepard. However, the latter uses have almost always given up AXIOM per se, although retaining the "ratio of strengths" formulation which provides the normalization. This and other facets of the similarity choice model, its properties, and relationships to other models of identification were explored in a paper by Landon and Townsend, "An experimental and theoretical investigation of the constant ratio rule and other models of visual letter recognition" *Journal of Mathematical Psychology* (1982). Rob Nosofsky and Doug Medin, to name two major theoretical figures in the field of categorization were instrumental in developing models of categorization and general classification based on the Luce-Shepard choice theory.

These days, mathematical modeling is increasingly carried out within rather complex, if cogent, quantitative theories using extraordinarily powerful computer software. This is all to the good and the field must and will evolve. Nonetheless, the kind of beautiful analytic modeling incorporating classical definitions-axioms-theorems-proofs-linkage-of-theorems-and-structure-to-experimentation, which can be seen and honored in so much of R. Duncan Luce's theorizing will remain part of scientific psychology and cognitive science.

The well-known summer institutes at Stanford University on mathematical psychology united a cadre of now famous psychological scientists including Richard Atkinson, Robert Bush, Bill Estes, Eugene Galanter, Duncan Luce, Patrick Suppes, and so many others. These institutes,

started in the 1950s and carried into the 1960's, helped develop the many seminal publications on the applications of probability to psychological theorizing. Many edited books brought to an excited graduate student audience the most recent works in the field. In 1960 Luce edited "*Developments in Mathematical Psychology*", and with Bush and Galanter in 1963 and 1965 *Handbook of Mathematical Psychology* Vols. 1, 2 and 3 and the two volumes of *Readings in Mathematical Psychology* in 1963 and 1965.

The discussions that occurred during these summer meetings set the stage for the development of new ideas about measurement. Classical measurement theory dates back to Helmoltz (1887) and Hölder (1901). Its application to the measurement of physical quantities is based on the idea that there are an observable ordering relation and an observable binary operation on the objects to be measured that satisfy certain testable axioms. The most important of such axioms say that a binary operation is commutative and associative.

In the first half of the twentieth century these ideas of classical measurement theory dominated measurement theory. Measurement theorists considered as unsound the methods of psychological measurement that were beyond simple ordinal measurement, because such methods were not implemented through an observable commutative and associative operation. In response S. S. Stevens formulated his famous theory of measurement in *Science* in 1948. In the 1950's, he and his students systematically applied his theory, particularly to psychophysical phenomena. This was the state of the scientific measurement environment when Luce began his work on measurement.

Luce's 1956 article "Semiorders and a theory of utility discrimination" was a forerunner to a new theory of measurement called "the representational theory of measurement" that Luce and Patrick Suppes later developed. Luce and Tukey's seminal article "Simultaneous conjoint measurement: A new type of fundamental measurement" appeared as the first article in the new *Journal of Mathematical Psychology* and demonstrated that some psychological qualities can be measured in a manner just as rigorously as qualities in the physical sciences.

They considered the situation of an ordering relation $\geq$ on a Cartesian product $X \times Y$ so that X can be measured by an interval scale ψ_1 and Y can be measured by an interval scale ψ_2 such that for all (x, y) in $X \times Y$,

$$(x, y) \geq (u, v) \text{ iff } \psi_1(x) + \psi_2(y) \geq \psi_1(u) + \psi_2(v).$$

Luce and Tukey provided conditions about how X and Y observably interacted with each other in terms of $\geq$ so that the interaction induced a commutative and associative operation.

The idea of using observable structure to define a commutative and associative operation that may not be directly observable, and using classical measurement with that operation to measure observable qualities and reinterpret the results in terms of the observable structure, became the key concept in Volume I and part of Volume II of Krantz, Luce, Suppes, and Tversky's seminal work, *Foundations of Measurement*. Eventually three volumes totaling 1426 pages appeared. These are considered a milestone of twentieth century science.

Luce and Narens generalized the key ideas of Vols. I and II of *Foundations of Measurement* so that the induced operation could be non-commutative or non-associative. They called their theory "non-additive measurement theory." Because they had no scientific application of it, they published it in the *Journal of Pure and Applied Mathematics* in 1976. Over a ten year period, Luce and Narens and students worked on this project. Finally in 1985 all the important pieces of the theory were developed and Luce and Narens published a 72 page summary in the *Journal of Mathematical Psychology*. At the end of that article they included an application to utility theory that provided alternative psychological explanations to some key phenomena of Kahneman's and Tversky's prospect theory. The research on non-additive measurement theory is summarized in Volume III of *Foundations of Measurement.* Luce (2000)

employed non-additive measurement theory as the basis for his theoretical and experimental masterpiece *Utility of Gains and Losses: Measurement-Theoretical and Experimental Approaches*.

Duncan's other books are marvels of scientific accomplishment. The early work with Raiffa, *Games and Decisions* (19570, the very famous *Individual Choice Behavior* (1959), the series of three *Foundations of Measurement* volumes with Suppes, Krantz and Tversky (1971-1990), *Response Times* (1986), *Sound and Hearing* (1993) and the just mentioned *Utility of Gains and Losses* are great works deserving the attention of students and scientists alike.

As a leader in theoretical developments Duncan also became a leader in the political development of scientific psychology. From 1964-1968 and 1971-74 he was a member of the Mathematical Social Science Board where he served as Chairman 1966-68 and 1972-74. He was elected to the National Academy of Sciences in 1972 and served in various capacities during the 1980's. He was a Fellow or Member of such prestigious scientific societies as the American Association for the Advancement of Science, the American Psychological Society, Federation of Behavioral, Psychological, and Cognitive Sciences where he served as Vice President, 1984-87 and President, 1988-91. He was also a member of the Psychometric Society serving as President, 1976-77, a member of the Psychonomic Society and its Representative to Federation of Behavioral, Psychological, and Cognitive Sciences, 1983-85. As a member of the Society for Judgment/Decision Making he served on the Executive Committee, 1987-90. One of the Societies he helped found and sponsored was the Society for Mathematical Psychology where he served on the Executive Committee, 1978-80, and as President, 1979.

His awards and honors speak to a career of accomplishment and distinction. He was elected to the Society of Experimental Psychologists in 1963, the American Academy of Arts and Science in 1966, he received the distinguished Scientific Contributions Award from the American Psychological Association in 1970. In 1972 he joined the National Academy of Sciences and in 1986 received the American Association for the Advancement of Science's Prize for Behavioral Science Research. In 1994 he was elected to the distinguished American Philosophical Society and in 2001 received the Gold Medal Award for Life Achievement in the Science of Psychology from the American Psychological Foundation. He was awarded the National Medal of Science in 2003 and President G. W. Bush presented the award to him in 2005. In 2004 Duncan received the Society of Experimental Psychologists Norman Anderson Award for Lifetime Contributions to Psychology, and in 2012 he received the Patrick Suppes Award in Psychology from the American Philosophical Society.

Here was a breathtaking career peopled by great scientists advancing ideas about psychology, economics, and scientific measurement itself. At the same time Duncan remained readily available to students and colleagues alike. Always an advocate for excellence, a strong voice for scientific psychology and a willing collaborator with a delightful wit, he was a model of decorum, a man of genius whose great works inspire others to continue his view of a scientific psychology.

Stephen Link
University of California, San Diego

Louis Narens
University of California, Irvine

James Townsend
Indiana University