ARISTOTLE (c. 384-322 BC) AND SIZE-DISTANCE INVARIANCE

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Abstract

In his book on Memory, Aristotle takes the comparison of the sizes of objects at different distances as an analogy of the comparison of time periods. He seems to be referring to the theory of size-distance invariance – that we take account of distance in a kind of geometrical manner when judging size. He proposed that there was a mental representation of the outside world, as distinct from a knowledge of distance brought by the travelling visual ray. He held to a concept of geometry that was developed from Pythagoras onwards and flourished later among the Stoics. This may be the earliest known reference to size-distance invariance.

Background to size-distance invariance

Size-distance invariance (SDI) is the idea that we perceive the linear size of objects by scaling the image size in proportion to the perceived distance. The ratio of an object's perceived linear size to its perceived distance is determined by the ratio of its real linear size to its real distance (which is equivalent to the object's real angular size). This is a geometrical construction of perception which has ancient origins. This view was supported by the Stoic philosophers, in opposition to the Epicureans. Epicurus (c. 341-270 BC) took the view that veridical perceptual knowledge was directly impressed on the sense organs by stimulus objects. He and his followers believed that vision was based on intromission – something entered the eye. The Stoics generally believed in extramission, or in a mixture of extramission and intromission – sensitive visual rays, or a cone of flux, went out from the eye and touched objects, and then brought back information (such as distance) to the eye. Influential Stoic writers who explicitly stated SDI were Posidonius (c. 135-51 BC), and Cleomedes (c. 2nd century AD) (Schönbeck, 1998; H.E. Ross, 2000). The astronomer, mathematician and optical theorist Ptolemy (c. 140 AD) also did so (*Optics* II, 56, Transl. Smith, 1996), and gave the diagram shown as Figure 1. The angle at E can represent the line AB at distance EB or GD at distance ED.

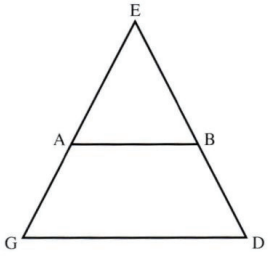

Figure 1. Ptolemy's diagram for size-distance invariance

It is sometimes said that the geometrical account arose after the Epicurean account, because it was linked to the Stoics mentioned above. However, Euclid (c. 300 BC) was a contemporary of Epicurus and he discussed the role of geometry in calculating sizes on the basis of different distances. He drew on the work of previous mathematicians, such as Pythagoras (c. 570-495 BC) or even Thales (c. 600 BC). As a perception theorist, he stated that perceived size followed image size (angular size), but that linear size could be calculated from a knowledge of angular size and distance. He wrote in his *Optics* (Theorem 5): "Objects of equal size unequally distant appear unequal and the one lying nearer to the eye always appears larger" (Transl. Burton, 1945, p. 358). In this passage Euclid used the language of *appearances.* In another passage (Theorem 21 - To know how great is a given length') he argued that linear size (true object size) could be calculated in a geometrical manner from the angular size and the distance: but in this passage he used the language of *calculation* rather than appearances. This is not quite the same thing as SDI, which states that perceived size equals perceived linear size, and follows automatically from perceived distance and the true angular size. The difference between the two accounts depends on the level of consciousness at which the size scaling is said to take place (H.E. Ross, 2003).

The passage in Aristotle's *De Memoria et Reminiscentia*

There is a passage by Aristotle (c. 384-322 BC) which seems to imply that the concept of SDI was already around before the time of Euclid. It occurs in a book on Memory (*De Memoria et Reminiscentia - 452b7-22*), rather than in the more appropriate *De Anima* (On the Soul) or *De Sensu et Sensibilibus* (On the Senses and Perception). However, memory is closely allied to both sense perception and thought, because material for memory is presented to the mind through the "common sense" (whose organ is the heart) - where sensations from the five special senses are integrated (Murray, 1988). The passage's location in a section on memory may be why it has been overlooked by psychologists studying perception. We came across it by accident, because the translation included a diagram resembling SDI which caught our attention when looking for material on the moon illusion. This diagram (Figure 2) is reproduced from *De Memoria* (in *Parva Naturalia*, transl. J.L.Beare and G.R.T.Ross, 1931). The diagram was reconstructed from the Greek text by J.L. Beare and his editor W.D. Ross, working independently, according to Sorabji (1994) and Sisko (1997).

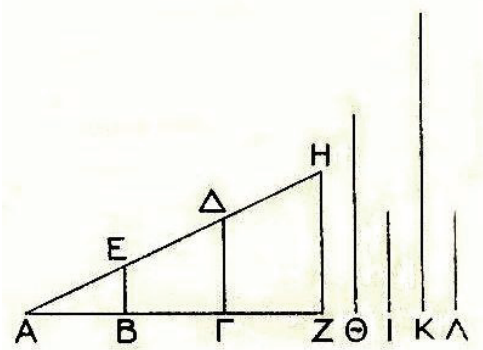

Figure 2. The diagram in Beare, J.I. and Ross, G.R.T (1931)

The text by Aristotle is ponderous and obscure, as are its translations. The various Greek manuscripts contain slightly different texts and alphabetic letters, and some contain diagrams. The translators choose a diagram to suit their interpretation of the text. Bloch (2007) thinks the manuscripts are too confused to give any diagram at all. Most recent translators give a diagram similar to Figure 2, with four lines shown beside the main triangle, but use the Roman alphabet instead of the Greek alphabet. All manuscript diagrams, and those of some of the earlier translators, show outer and inner triangles, corresponding to time and space experience respectively. This is illustrated in Figure 3, for the diagram of Freudenthal (1869). As Freudenthal (p. 417) explains, Aristotle sometimes used one letter to denote a whole line. In Figure 3 this is indicated by the curves H (running from M to H) and M (running from M to K). Freudenthal interprets BE as an inner representation of space, and KL as an inner represention of time. In this way he devises a kind of personal measuring stick which is independent of objective space and time. He may be hinting at SDI for spatial perception.

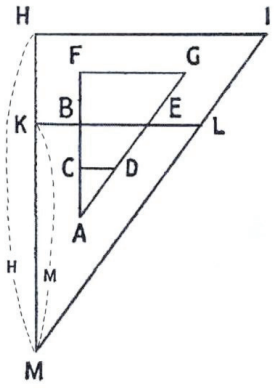

Figure 3. The diagram in Freudenthal (1869) with lettering by Hammond (1902)

We give our own translation below, to correspond to the Freudenthal diagram in Figure 3. The words in square brackets are added as our interpretation, but the material in round brackets is in the original text.

"But there is a most important fact to be noticed—that we must have apprehension of time either determinate or indeterminate. Let us grant as real something by which we discriminate greater and less periods. It is reasonable that we should do so in the same way as we discriminate extended magnitudes; we know things that have great size and are at a distance, not by our thought reaching out to them there, as some say our sight does (for non-existent things can equally be known), but by a proportional movement: for there exist in our thought figures and movements similar to the external objects. What then is the difference between thinking the objects of greater size and thinking the objects of smaller size? All the inner [representations] are smaller and the outer [objects] are proportionate; but it might well be that, just as in the case of the spatial things, the person has another corresponding representation within him, so it is with intervals. Thus, if one makes the movement AB, BE, this produces AC, CD (for AC and CD are in the identical ratio as AB and BE). What is it that makes AC, CD rather than AF, FG? Or, just as [with spatial objects] AF is to AB, thus [corresponding with time] H [MH] is to M [MK]. These processes, then, move together, but, if one wants to think [the external object] FG, one equally thinks [the corresponding representation] BE.Instead of the [time objective] ratio of H [MH] and I [HI] one thinks that inner [time representation] K [MK] to L [KL], for these [time objective MH and internal representation MK] are in the same proportion as [the object] FA stands to [the representation] BA."

Some authors give a diagram similar to that of G.R.T. Ross (1973, p. 115), which we show as Figure 4. The diagram is essentially the same as that of Beare and Ross (1931), apart from the difference in orientation and the use of the Roman alphabet. G.R.T. Ross (1973 p. 279) credits W.D. Ross with this diagram. He contends that Aristotle's aim was to show that external distances and movements are reproduced in miniature in the mental organ; the internal representations are proportional to the external realities, just as the sides of a small triangle are proportional to those of a much larger one

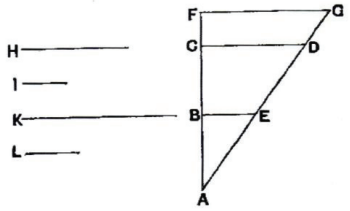

Figure 4. The diagram in Ross, G.R.T. (1973).

Roark (2011, p. 156-157) rejects the above interpretation. He argues that Aristotle was not concerned with the difference between external and internal magnitudes, but with the question of how differences in external magnitudes are represented. He maintains that, in Figure 4, the lines BE, CD and FG all have the same 'apparent' size because they subtend the same angle to the viewer at A; so how can the sizes be distinguished? Roark says that the key to answering this question is Aristotle's remark (in his translation) that "as we may assume within a person something proportionate to the forms of things, so we may equally well assume something else proportionate to their distances." (This is the part that we translate as "it might well be that, just as in the case of the spatial things, the person has another corresponding representation within him, so it is with intervals.") Roark then argues that Aristotle intended some kind of scaling of size according to distance, so that we perceive or imagine true object size rather than angular size. However, he makes no mention of the literature on SDI. He goes on to argue (p. 162-163) that this interpretation is consistent with some of Aristotle's other remarks about size. Aristotle says (*De Anima* 428b29-30) that our perception of the common sensibles is liable to error "especially when the perceived object is at a distance". For example, Aristotle says in at least three places that the sun appears to be one foot in diameter (*De Anima* 428b2-4; *De Insomniis* 458b28-9, 460b18-19). Aristotle wrote (*De Anima* 428b2-4) "But things also appear falsely, when one has at the same time a true supposition about them (e.g. the sun appears a foot across, but is believed to be bigger than the inhabited world)" (Transl. Schofield, 1978). Roark argues that we perceive the sun as small because we have no experience of seeing it at different distances, and it is vastly further away than any distances we see on earth. He adds that comparison with other distant celestial objects is of little help, and that the size of nearer occluding objects (e.g. mountains, clouds, the moon) gives only a minimal size to the sun. Thus, in the case of the sun, we do not have the ability to scale for distance. Again, Roark makes no mention of the large literature on size perception and the causes of the sun or moon illusion (e.g. H.E. Ross & Plug, 2002).

Sisko (1997) and Sorabji (2004. p.18) use the diagram of Beare and Ross shown in Figure 2. Sisko says that Aristotle probably held the view that the perception of magnitude by sight involves the production of small-scale analogues within the mind. Sorabji agrees that the diagram is concerned with judging the relative sizes of larger and smaller objects, and thinks that the mental diagram is more useful for calculating spatial distances than for calculating temporal ones (p. 108-109).

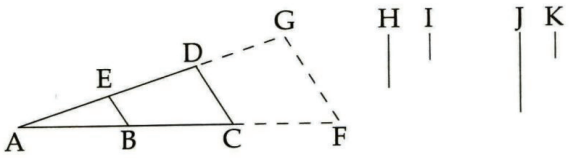

Figure 5. The diagram in Sachs (2004)

Sachs (2004, p. 177-178) gives a broadly similar translation and diagram to other recent authors. His diagram (Figure 5) is similar to that of D. Ross (1955, p. 250). He says that the appropriate diagram needs to be of the same kind as the one accompanying Proposition 12 in Book VI of Euclid's *Elements*. However, this proposition is about the geometry of proportional lines, and not about size and distance. There is of course a difficulty in saying how 'size' and 'distance' differ from 'length', and this remains a difficulty for modern versions of SDI (e.g. Schwartz, 1994). Aristotle's concept of 'movement' may well deal with 'distance' rather than 'length'. Sachs takes a different interpretation of the diagram from Roark, Sisko and Sorabji, saying "These lines are not meant to be representational images of any content in the imagination, but play the same role the lines do in the 'universal mathematics' of Book V of Euclid's *Elements*." Though he implies that the diagram is not about size scaling, he does say that it is similar to diagrams of Euclid. He points out that the 'something by which one distinguishes the time' is the common or primary perceiving power.

It is not obvious how the comparison of time periods is similar to the comparison of sizes and distances, though most of the commentators mentioned above discuss the issue. However, modern research shows that the ability to discriminate differences in time generally obeys Weber's law (Wearden and Lejeune, 2008).

Conclusions

The Aristotle manuscripts are indeed confused and hard to interpret. However, the interpretations of Freudenthal, Sisko, Sorabji and Roark are plausible, and they lend support to the idea that Aristotle is referring to the notion of SDI. In essence, Aristotle is saying that the comparison of periods of time is like the comparison of spatial sizes; we compare the latter in a geometrical manner, by taking account of distance, and keeping the ratios of sizes proportional to that of the distances. All of this takes place in 'thought'. In this passage Aristotle proposes a kind of mental gymnastics, as distinct from the work being done by an outgoing travelling ray (a view held by Plato and Empedocles – and elsewhere by Aristotle). He is thus quite modern in his idea that the mind (brain) creates a representation of the outside world. His description of spatial perception seems to be closer to that of perceptual SDI than to the mathematical calculations of Euclid. He does not say that perceived size is equivalent to mathematical angular size, but rather that it is equivalent to a range of linear sizes depending on the perceived distance. If this is a correct interpretation, it means that the idea of SDI was around much earlier than is usually thought.

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