FUNCTIONAL RELATIONS AND CAUSALITY IN FECHNER AND MACH

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Abstract

In the foundations of Fechner's psychophysics, the concept of "functional relation" plays a highly relevant role in three different respects: (1) in respect to the principles of measurement, (2) in respect to the mind-body problem, and (3) in respect to the concept of a law of nature. In all three cases, it is important to explain the difference between the functional dependency of one variable upon another, on the one hand, and the causal relationship between variables on the other. In all three respects, Ernst Mach developed Fechner's ideas further and tried to extend the lessons of psychophysics learned through the concept of a functional relation to the whole of science. For all three cases, I try to indicate why they are still relevant for psychophysics today.

In his *Elemente*, Fechner (1801-1887) defines psychophysics as an "exact doctrine on the functional correspondence or interdependence of body and soul." "Functional correspondence" [*funktionelle Abhängigkeitsbeziehung*] is then defined as a "constant or lawful relation between both [the material and the mental] such that we can infer from the existence and the changes of one the existence and changes of the other." (1860, 1:8) Fechner makes it clear that such a relation is called "functional", because it states the dependency of a psychological variable on a physical one (or the other way around) in the same way as a mathematical function describes a dependency relation between *x* and *y*.

By using definitions like these, the nature of psychophysics is intrinsically connected with the concept of a mathematical function in a considerable and important way. The question then arises why Fechner links up the real meaning of his new science so closely with the form of a mathematical function. I do not think it is only because Fechner wants psychophysics to be a mathematically formulated science. No one in the history of science prior to Fechner seems to have defined a scientific discipline in relation to the functional dependence of one variable upon another.

A first glimpse of such a move before Fechner can be made out in the preface of Emil Du Bois-Reymond's 1848 work on electrophysiology, in which he advocates the ideas of the biophysical movement of his time. He says there that the seeds of a "physico-mathematical" view of physiology lie in the aspiration "to imagine the causal connection of natural phenomena [in physiology] by the mathematical picture of dependency, by a function." Without such a view, Du Bois-Reymond continues, "even the most exact determinations of measurement would be as fruitless for understanding life-processes as the mere measurement of a machine's dimension would be for comprehending its workings." And later on he adds that the "true challenge for physiology is not so much to detect the causes" of mechanical movement of matter to which the physiological phenomena reduce in the end, "but to find the laws by which they are governed." (1848, 6 & 15)

Fechner seems to have conceived of psychophysics in a similar way. He gives two reasons—both of which are very similar to those given by Du Bois-Reymond—why the psychophysical relation is to be treated and conceived as a mathematical function:

1. Doing so makes it possible to avoid any reference to causality. The relation between dependent and independent variables in a mathematical function does not in itself say anything about the causal meaning of this relation, nor about any asymmetry relevant for distinguishing between cause and effect. A mathematical function is itself completely neutral with respect to causality.
2. Doing so makes possible the measurement of the psychological and thereby a deeper understanding of psychology: "Only the physical can be measured directly whereas the measure of the psychical can be accomplished only in dependence on it [i.e. on the physical]." (Fechner 1860, 1:9) There is therefore an asymmetry in the relation between the psychical and physical which can adequately be expressed only by a mathematical function. Hence, outer psychophysics has to concentrate on the functional dependency of the psychological on the physical and not on the inverse dependency. (Fechner 1851, 203)

But why should we avoid causality and how does a functional relation help us to do so? How does a functional relation make measurement possible? Let us have a closer look at the three areas of Fechner's thought where the functional relation plays an important role.

Functional relations and principles of measurement

As Fechner explains, an instrument A measures a dependent variable Q by R_A only if

(a) every value of Q serving as input to A is associated with a unique value of R_A that is realized in A, and

(b) the order of values of Q in A is such that it uniquely correlates to an order of values of R_A in A.

If a measurement apparatus A fulfills these properties, then R_A functions as a *representative* of Q, and A realizes a *constant correlation* between Q and R_A.

Take for example a mercury thermometer that measures the temperature Q of a liquid (independent variable) in virtue of the length R_A of the mercury column (dependent variable). We can infer from the length of the column the temperature of the measured liquid because the behavior of mercury serves as a representative of the state of temperature. The thermometer realizes a constant correlation between the temperature and the independent variable R_A such that the height of the mercury column correlates with the size of the temperature. If we calibrate the thermometer in the right way, we can express the values of the temperature quantitatively.

In order to have a simple and easily manageable measuring instrument, we additionally choose or construct a representative and its calibration in such a way that the quantitative values of Q and R_A can be made equal. If we call the equation between the values of Q and those of R_A a "measurement formula", then the most convenient and useful measurement formula in the thermometer case would be $Q = n \cdot R_A$, with $n = 1$.

Can the conditions stated above be taken as sufficient for measurement? Not yet, because, as Fechner argues, we have to be sure that the equality of values of Q can be determined empirically, and we have to know when Q is zero. (Fechner 1851, 203; 1860 1:58f., 63f.; 1887, 213) So we have to add a further necessary condition to the above definition if it is to be sufficient: (c) It must be empirically possible to determine differences between values of Q to be equal, and this equality must be preserved by the measurement formula. If (c) were not added, the choice of the measurement formula would be completely arbitrary. Note that the measurement formula does not state that a certain empirical *law* holds between Q and R_A. Rather, it describes how Q and R are related to each other by the measuring instrument that was chosen or constructed for the measurement of Q. The measurement formula is thus a conventionally and freely chosen *functional relation* and is as such to be distinguished from a *law of nature*. It is true that the choice among all possible conventions is *guided* by empirical facts, but this does not make it a natural law.

In Fechner's time, the objection was raised that R_A must be caused by Q in order to be measurable. (Elsas 1886) Thus the thermometer measures temperature because the temperature causes the mercury column to expand. Fechner answered that a functional relation in physics might also represent a causal relation, but that being causally related is not necessary. The controversy thus boils down to the question whether condition (c) is really feasible.

Let us now consider how Fechner transfers this reasoning from the case of physics to the problem of measuring sensations. To measure sensations we first have to find or construct a suitable instrument for which we know the measurement formula involved. The only variable from which we know that it varies with sensation S and which therefore is the only candidate to serve as a representative of S in a measuring instrument seems to be the physical intensity I of the stimulus. The only device which could possibly serve as a measurement apparatus relating S to I is the living human body. So instead of being able to choose from an array of possible candidates as measuring instruments for sensations, we are limited in our world to a single one. We find ourselves, as it were, in the position of someone who finds a physical measuring instrument fabricated by an alien culture, for which the measured variable and its representative are known, but not the relevant measurement formula.

Now, the task of outer psychophysics is precisely to find the measurement formula. It follows from Weber's law that the measurement formula for mental measurement cannot be as simple as in the case of normal physical measurement. As we all know, Fechner eventually arrives at his logarithmic solution $S = k \cdot log\ I / I_0$, where I_0 is the absolute threshold. This means that we have to determine I_0, and also the stimulus increment necessary to produce a just noticeable difference when added to the starting stimulus level I_0.

Fechner claims the validity of the logarithmic formula also, indeed first and foremost, for inner psychophysics—this time not as a measurement formula, but as a fundamental empirical law of nature. This law expresses the relationship of psychophysical activity and the subjective dimension. Inner psychophysics is concerned with the relation of "psychophysical activity" or "excitation"—i.e., of the nervous activity of the brain parts immediately connected with the mind—to subjective phenomena. So Fechner claims that if we take the living human body as a measuring instrument of sensations, the values of S found in outer psychophysics are logarithmically related to the fundamental neural activity. It is thus only a coincidence that the same formula figures in outer as well as in inner psychophysics. If in a different world Weber's law looked different, the fundamental law could very well differ from the measurement formula.

Functional relations and the mind-body problem

The second area of Fechner's psychophysics for which the difference between "law of nature" and "functional relation" is important is mind-body theory. We have seen that Fechner defines psychophysics as the doctrine of the functional dependence of body and soul. This is now meant in a different sense than discussed above. It means that psychophysics should investigate only the "phenomenal side of the material and the mental world" and refrain from saying anything about the essences behind these phenomena and their relation. (1860, 1:8) It would be metaphysical, for example, to regard the relation between mental and material phenomena as causal or as constituting evidence supporting the identification of mental states with brain states. This means that psychophysics should avoid any mention of objects (including the “soul”) and of causal relations between them. To go beyond the world of phenomena always involves a metaphysical commitment that should be avoided.

Fechner is not against metaphysics as a matter of principle, but against *premature* metaphysics. Only a mature science can suggest a plausible metaphysical complement for its starting principles, but a young science like psychophysics ought to refrain from such an attempt for a long time yet, just as physics had to wait quite long before atomism became plausible. Psychophysics turns into metaphysics as soon as one asserts something “about the reason [for a functional dependency], about its interpretation and its scope”, that goes beyond the actually given relations. Fechner obviously wants to avoid any realistic assumption about the nature of mind and body and the relation between them. It seems that he distinguishes two components of a realistic statement about the mind-body relation: a core element stating a functional relation between a material and a mental variable, and an additional component that interprets this relation in a realist way, as is done, for example, by Cartesian mind-body dualism or identity theory.

Fechner maintains as a basic principle of psychophysics that every mental event has a “psychophysical substrate”—a “support” or “basis” in the brain. This conception has survived until today in the idea of the “neural correlate” of the mental. A neural correlate of mental activity is the minimal set of neurons, whose firing directly correlates with the relevant mental activity of the subject at a given time. Conversely, stimulating these neurons in the right manner with some yet unheard of technology should give rise to the same mental activity as before. (Koch 2004, 16) Fechner and today’s neurophysiology agree with each other that the investigation of the underlying substrate, whether it is called a “neural correlate” or “psychophysical activity”, does not by itself commit one to a particular metaphysical view about the relation between neural matter and the mind. This is why Fechner calls the relation between a mental activity and its neural correlate a “functional relation” and avoids any causal terminology in its characterization. We can observe the Fechnerian heritage even today: psychophysics is still a science concentrating on functional dependencies and avoiding talk of causal relations.

Functional relations and laws of nature

Fechner’s peculiar distinction in psychophysics between a functional relation and a natural law implies a special view of the notion of a law that is at variance with a widely popular view today. In the greater part of today’s philosophical literature, the paradigm of a law of nature is a conditional proposition like: “If the wire is heated, then it expands;” i.e., it is taken as a relation between events or conditions. Consequently, one has to make an interpretative effort in order to show that a law like the 2nd law of Newtonian mechanics, $F = m \cdot a$, is only a different formulation of such a conditional proposition. For Fechner, however, the paradigm of a law would be precisely such an equation. Consequently, he is obliged to show how a conditional proposition can be reduced to such a form.

There is, however, a recent exception to mainstream philosophy of science that comes close to Fechner’s idea. Herbert A. Simon and Nicholas Rescher have shown that, given a system of functional relations with certain properties and a set of variables appearing in these equations, an asymmetric relation can be introduced among individual equations and variables that corresponds to the notion of a causal ordering. (Simon & Rescher 1966) The crucial role is thereby played by the concept of a self-contained structure of function. Fechner would certainly have liked this result because it shows that a ‘thick’ structure of purely functional relations can induce a causal order among the variables. It can be shown that a set of functions that is not yet self-contained—i.e. roughly, that has as many functions as variables—cannot yet determine causal relations among the variables.

Functional relations in the hands of Ernst Mach

I now want to show how Ernst Mach tried to develop a whole new conception of science by transforming Fechner's methodological precautions into general scientific principles. His basic idea was that it is not psychophysics that needs reinterpretation in order to harmonize with normal scientific practice in physics, as many contemporaries of Fechner wanted, but that philosophy of science must change so that it suits psychophysics! And neither must psychology be built on physics in order to make sense but, rather, physics should wherever possible be erected upon the foundations of psychophysics.

According to Mach, the task for physical measurement is to find a suitable representative for sensations that can be used in a functional relation. We know through experience that the expansion of volume can serve as an indicator of our heat sensations. However, as it is the goal of physics to relate volume also to other phenomena in the world, it will come to deviate from the first goal and try to find a representative of the temperature scale that allows us to make simple, productive and clearly arranged claims about the relations existing between volume and other physical phenomena. The deviation from the original functional relation is minimized as much as possible. If we are clever enough in selecting the right indicator for heat, this will enable us to better understand the relations of things in the external world and to make predictions about heat phenomena that are superior to those we would get sticking with representing pure heat sensation. Another reason for changing the initial choice of a representative of sensation lies in the limited reliability of our bodily constitution. Instead of using our individual hands and feet to determine spatial extension without perceiving spatial changes in them, we select a publicly accessible standard that is more rigid. (Mach 1886, 157)

This view of things allowed Mach to solve some problems he had diagnosed in physics. First of all, measuring is not a matter of discovering an intrinsic property of an object, but of discovering the relation existing between the thing measured and the freely chosen standard of measurement. It is therefore idle to look for the 'true' measure of a magnitude. The fact that we often think we must find an objective length, an objective time-period or a true temperature must be explained by a psychological mechanism: since physics very often replaces the indicator of a sensation by a representative that favors the relations of external objects among each other in such a way that the latter and the former are not entirely parallel, "we secretly and unconsciously harbor a notion of the original sensation as the *core* of our ideas." (Mach 1896, 51) This "shadowy core" then acquires the imagined role of being the 'real' magnitude which is at most approximated by the indications of a measuring instrument, but never exactly portrayed. The notions of absolute space and time are such relics of the pre-physical perception of space and time prior to the institution of clocks and rulers. "Newton's notion of 'absolute time,' 'absolute space,' and so forth [...] originated analogously. In our ideas of time, s*ensations of duration* play the same part for variously measured portions of time as the sensation of heat does [for measured temperatures in our idea of heat]. The situation is similar for space." (Mach 1896, 52) –

As is well known, Fechner favored a dual-aspect (or better: a double-perspective) theory as a solution to the mind-body problem. The living human body can be perceived from two perspectives: from the outside, if seen by another person, or from the inside, if perceived by the person herself. In order to clarify the relation of Mach's theory to Fechner's solution, one has to distinguish between the psychophysicist's renunciation of a causal interpretation of the psychophysical law and the rejection of causality by the mind-body philosopher trying to spell out the most plausible solution to the mind-body problem in view of the progress of psychophysics. In other words, one should tease apart the methodological postulate that psychophysics ought to treat the mind-body relation as a functional relation from the noncausal interpretation of the psychophysical law in philosophy. Fechner's *philosophical* doctrine is the result of just such an interpretation. For him, mind and body are related to each other like the two sides of a coin, which cannot be said to have a causal relation between them, but are aspects of one and the same object. In order to avoid any reference to an unknown reality that is neither material nor mental, as in Spinoza, Fechner rejected the concept of substance from very early on and conceived of the living human body as a bundle of lawfully connected appearances. (When, in 1886, Mach rejected Fechner's view, he was criticizing, as he admitted in a later note, Fechner's pre-1855 viewpoint, when Fechner had not yet argued for the bundle theory of substance. For more details see Heidelberger 2000.)

In 1886, Mach rejected Fechner's notion of perspective and wrote that "the elements given in experience, whose connection we are investigating, are always the same, and are of only one nature, though they appear, according to the nature of the connection, at one moment as physical and at another as psychical elements." (Mach 1886, 51) This means that Mach replaces the two different perspectives by two kinds of functional dependency that can obtain between the elements. This is the basic idea of neutral monism. It might be possible that Mach's reason for this is to avoid any remnants of an irreducible ego that might be implied by the notion of a perspective of a person toward herself. Surprisingly enough, Mach returned again to Fechner's mature theory in 1905. –

We have seen that for Fechner, psychophysics turns into metaphysics as soon as one surmises about objects (including the soul) that are behind the psychophysical appearances. Instead of working out criteria for when such an inductive metaphysics is scientifically acceptable and plausible, Mach rejects such an outlook not only for psychophysics, but for science in general. This means not only that physics has to forego atoms but that the concept of causality has to go as well, and that functional relation must take its place: "The old-fashioned idea of causality", Mach writes, "is a little clumsy: A dose of cause is followed by a dose of effect. This represents a kind of primitive, pharmaceutical *Weltanschauung*, like in the doctrine of the four elements. [...] The connections in nature are rarely ever so simple that one can identify *one* cause and *one* effect. Consequently, I have tried for a long time to replace the concept of cause by the mathematical concept of the functional relation: dependency of the appearances from each other." (Mach 1886, 74) We can see how fruitful Fechner's distinction between a functional relation and a (causal) law has become.

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