

# MEASURING MEMORY PROCESSES THE POINT OF SUBJECTIVE EQUALITY AND THE CONSTANT ERROR

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## Abstract

*A theory of memory retrieval is created that predicts shifts in the Point of Subjective Equality due to the cost of retrieving a standard stimulus magnitude from memory. The theory provides a method of estimating the cost of memory retrieval in units of the physical stimulus. Tests of the theory are provided by using a very early classic experiment from Werner Brown (1910).*

We feel our sensations. The immediacy, vividness, clarity, and sharpness of our feelings lend a precision to our sensations that suggests accurateness and reliability. Yet even the earliest psychophysical experiments produced laments about the failures of experimental subjects to conform to such expectations. Fechner (pp. 90-91, 1860/pp. 75-76, 1966) comments,

When the method of right and wrong cases is used in weight-lifting experiments, the constant error is demonstrated when a large number of cases where the container with the comparison weight was lifted first is compared with an equally large number of cases where it was lifted second, while all other circumstances were the same. The ratio of right to wrong cases in the one instance will be quite different from the ratio in the other. ...I must admit that this quite unexpected occurrence of constant errors in these experiments was most puzzling to me ....

The unexpected occurrence of “constant errors” sometimes called Time-Order Errors (TOE) or Space Order Errors was hardly limited to Fechner’s experiments. By 1910 the psychometric function, created by Urban (1906), also revealed constant errors. But in the psychometric function the evidence for the “constant error” became even more profound. A reanalysis of Brown’s 1910 data, shown in Figure 1, reveals the bizarre paradox that occurs when two stimuli of identical magnitudes are compared against each other. The standard weight of 100g was always lifted first by the experimental subject. One expects the comparison weight of 100g to be judged heavier or lighter with equal probability of 0.50. Yet Figure 1 shows that the estimated probability of reporting the second weight of 100 grams to be heavier than the standard of 100g equals 0.89.

Furthermore, the Point of Subjective Equality, the value of the comparison stimulus generating responses of “heavier” with estimated probability 0.50, expected to occur at the comparison weight of 100g occurs instead at 95g. We will see that the 5g shift in the PSE to 95 g from its expected position at 100g is a measure of the constant error.

The many excellent investigations of the constant error (Hellström, 1979) laid the foundation for a broad range of possible explanations. While these hypotheses each merit attention, none predicts new phenomena, although many provide good fits to the data that gave rise to the hypothesis.

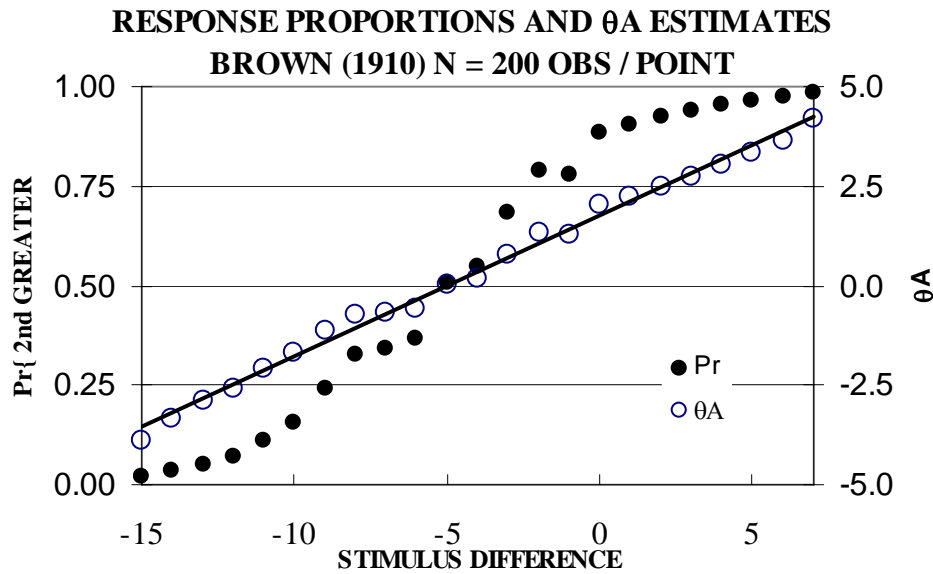


Fig 1. Psychometric function and values of  $\theta_A = \ln(p/(1-p))$  for heavier judgments with the Standard 100g weight always presented first. Experiment performed at University of California, Berkeley, Werner Brown (1910).

### An opponent process theory of memory retrieval

To postulate a source of the “Constant Error” is to develop a more extensive theory of comparative processes than those based solely on stimulus representations. In particular the role of memory processes in the storage and retrieval of stimulus representations will provide a new foundation for analysis of psychophysical experiments.

The addition of memory processes to the previous theory of judgments developed by Link (1975), Link and Heath (1975) and Link (1992) begins by including in the formulation processes for memory storage and retrieval. In particular let  $Po(\varphi)$  designate a Poisson distribution with parameter  $\varphi$  and for a time interval of size  $\Delta t$  define

$W_S(\Delta t) \sim Po(\mu)$  to be the image of a Standard stimulus  $S_S$  and

$W_C(\Delta t) \sim Po(\lambda)$  be the image of a Comparison stimulus  $S_C$  where  $\sim$  means “distributed as.”

In previous formulations a subject’s decision was based on the accumulation of momentary values of  $D(\Delta t) = W_C(\Delta t) - W_S(\Delta t)$  with  $E[D(\Delta t)] = E[W_C(\Delta t) - W_S(\Delta t)] = \lambda - \mu$  the expected comparative difference within time unit  $\Delta t$ . The random walk formulations introduced by Link (1975) and Link and Heath (1975) proposed that momentary comparative differences are accumulated until a response threshold is first exceeded.

Memory processes affecting the creation and storage of a stimulus image are important components in defining the stored image of a proximal stimulus. For the moment, however, I want to focus attention on the process of retrieval and its effects on the stored image of the Standard stimulus. In this regard let  $R(\Delta t) \sim Po(\rho)$  be the effect of retrieval of the Standard  $S_S$ .

That is, during the comparison process the immediately available Comparison stimulus will be judged against the Standard as affected by the process of retrieval.

In this regard let the process of comparison occurring during a unit of time  $\Delta t$  generate a discrimination statistic,

$$D(\Delta t) = W_C (\Delta t) - [W_S (\Delta t) - R (\Delta t)]. \quad (1)$$

The meaning of retrieval conveyed in Eq 1 is that of an opponent process extracting the stored stimulus image. The comparative process itself is characterized as an opponent process creating differences between the Comparison stimulus and the retrieved Standard.

As in previous formulations of the stochastic process of decision, the momentary values of  $D(\Delta t)$  are accumulated until a response threshold at either values  $A$  or  $-A$  is first exceeded. The value  $A$  is associated with a decision that the Standard is larger, heavier, bigger, or in general greater than the Comparison stimulus. The response threshold at  $-A$  is for the alternative decision that the Comparison stimulus is smaller, lighter, or in general less than the Standard stimulus. This process is easily extended to include response bias that begins the decision process with an initial value equal to  $C$ . These ideas are explained more fully in Link (1992) Link and Heath (1975) or Shedlan (2006).

These considerations lead to characterizing the decision process as a random walk beginning at a starting position  $C$  and terminating at response threshold values of  $A$  or  $-A$ . The probabilities responding "Heavier" or "Lighter" are obtained by using the Wald Identity. This expected value theorem leads to solutions for both response probabilities and decision times in units of  $\Delta t$ . Letting  $D$  be the value of this stochastic process when the process terminates provides for a Wald Identity expressed as the expected value of  $\exp(-\theta D)$  where  $\theta$  is the non-zero solution to

$$E[\exp(-\theta D(\Delta t))] = 1, \quad (2)$$

where  $\exp$  designates  $e$ , the base of the Napierian logarithm. Therefore,

$$E[\exp(-\theta W_C (\Delta t) + \theta W_S (\Delta t) - \theta R (\Delta t))] = 1. \quad (3)$$

Since each of the components of Eq 3 is Poisson distributed Eq 2 is also written as

$$\exp(\lambda \frac{\exp(-\theta) - 1}{\theta}) + \exp(\mu \frac{\exp(\theta) - 1}{\theta}) + \exp(\rho \frac{\exp(-\theta) - 1}{\theta}) = 1. \quad (4)$$

Solving Eq 4 for the non-zero value of  $\theta$  gives,

$$\theta = \ln((\lambda + \rho)/\mu). \quad (5)$$

Notice that the parameter for the Comparison stimulus can be represented as  $\lambda = \mu + \delta$  where  $\delta$  is the difference between the mean values for the Comparison stimulus and the Standard.

Thus,

$$\begin{aligned}\theta &= \ln((\mu + \delta + \rho)/\mu) \\ &= \ln((1 + \delta + \rho)/\mu)\end{aligned}$$

which may be approximated by

$$\theta \approx (\delta + \rho)/\mu. \quad (6)$$

To determine the probability  $p$  of responding “Heavier” the Wald Identity itself is evaluated at the values of  $A-C$  for the upper response threshold and  $A+C$  for responding “Lighter” at the lower response threshold. Because the Wald Identity is an Expected value theorem and only two possible values of  $D$  are possible, either  $A-C$  or  $A+C$ ,

$$E[e^{-\theta D}] = pe^{-\theta(A-C)} + (1-p)e^{-\theta(-(A+C))}, \quad (7)$$

which yields,

$$p = (1 - e^{-\theta A})/(e^{\theta A} - e^{-\theta A}) + (1 - e^{-\theta C})/(e^{\theta A} - e^{-\theta A}). \quad (8)$$

Thus the probability of responding “Heavier” is determined by two components. The first term on the right-hand side is free of response bias,  $C$ , while the second term of the right-hand side depends on response bias. If there is no response bias Eq 8 becomes,

$$p = 1/(1 + e^{-\theta A}), \quad (9)$$

the defining probability for a Logistic distribution function with parameter  $\theta A$ .

Also, when there is no response bias  $C = 0$  and Eq 9 may be used to provide an estimate of the unknown value of  $\theta A$ . In particular,

$$\ln(p/(1-p)) = \theta A. \quad (10)$$

By substituting for  $\theta$  in Eq 10 we have

$$\begin{aligned}\ln(p/(1-p)) &= ((\delta + \rho)/\mu)A \\ &= (\delta/\mu)A + (\rho/\mu)A \\ &= \delta(A/\mu) + \rho(A/\mu).\end{aligned} \quad (11)$$

### Predictions of the theory

Using the Newtonian idea that small theoretical changes can be represented as a similarity transformation of small physical changes suggests replacing the unknown value  $\delta$  by  $k\Delta S$  the physical difference between the Comparison Stimulus and the Standard multiplied by a

constant of proportionality. This substitution provides a testable consequence for this theory of retrieval from memory because the left-hand side of the equation may be estimated from response proportions and the right-hand side is linear in  $\Delta S$ . That is,

$$\ln(p/(1-p)) = k\Delta S(A/\mu) + \rho(A/\mu) \quad (12)$$

is a linear equation with slope  $k(A/\mu)$  and intercept  $\rho(A/\mu)$ .

This predicted linear equation is a test of the theory and provides for estimation of unknown parameters. The response proportions from Brown (1910) are shown by the closed circles in Fig 1. These proportions yielded the estimates of  $\ln(p/(1-p)) = \theta A$  shown by the open circles. The linearity shown by the solid line as a best fit to the estimates of  $\theta A$  as a function of  $\Delta S$  is unquestionable. The linear equation proves to be

$$\ln(p/(1-p)) = (0.35)\Delta S(\text{grams}) + 1.75. \quad (13)$$

Now the estimates of slope (0.35) and intercept (1.75) can be used to determine the unknown cost of retrieval from memory,  $\rho$ , because the intercept value  $\rho(A/\mu)$  divided by the slope value  $k(A/\mu)$  equals  $\rho/k$ . Using the estimated slope and intercept values provides an estimate of  $\rho = k5.0$  grams. Retrieval from memory costs 5 grams times the constant of proportionality.

As an additional observation, notice that the Point of Subjective Equality occurs for that value of the Comparison stimulus generating “Heavier” responses with probability 0.50. When  $p=0.50$  the left-hand side of Eq 12 equals zero. Solving Eq 12 for the value of  $\Delta S$  under this condition yields  $\Delta S = -\rho/k$ . Thus the Point of Subjective Equality is shifted 5 grams from the value of a Comparison stimulus equal to the Standard. This is another test of the theory that must follow from the linearity in Eq 12.

But the estimation of the cost of retrieval from memory is not the only aspect of the decision process to be estimable from this development. When the value of  $\delta$  equals 0 the right-hand side of Eq 11 equals  $\rho(A/\mu)$ . Therefore  $1.75 = \rho(A/\mu)$ . Substituting for  $\mu$  the value  $= kS_S$  gives  $\rho(A/kS_S) = (\rho/k)A/S_S$  or

$$\begin{aligned} A &= 1.75 S_S/(\rho/k) \\ &= 1.75 * 100\text{grams}/5\text{grams} = 35. \end{aligned} \quad (14)$$

Thus, the known value of  $\rho/k = 5$  and the Newtonian assumption regarding proportionality of the internal stimulus representation provides an estimate of  $A$ . While this value may seem quite large it applies to all comparison weight judgments used in this judgment process, weights ranging to 110g from 82 grams.

### Conclusion

This first measurement of a memory process in units of the physical stimulus is a complement to previous psychophysical measures of mental phenomena. As space is limited only the

beginnings of this development are presented here. Still, the explanation for the frequency of “constant errors” is now apparent from the fact that the first stimulus presented as a Standard stimulus must be stored in memory and then recalled when the second stimulus, the Comparison stimulus, is presented for judgment. Various experimental design features will influence the salience of the Standard, the availability of the Comparison, the duration between the offset of the Standard and the onset of the Comparison, all design features known to affect performance. Each of these must have its own effect in terms of opponent processes in the complete decision process.

Brown was unable to measure response times in the lifted weight experiments. But, in formulating a theory of the psychometric function, Link (1978) analyzed both response times and response proportions for a number of classic psychophysical experiments. The theory presented above, sans the memory retrieval component, gave an excellent account of all the data including predicted relations between response times and response proportions. But, the earlier theory could not predict the shift in the Point of Subjective Equality found in those earlier experiments. Those shifts in the Point of Subjective Equality and the shift in the peak of the Chronometric functions are correctly predicted from the theory created here.

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