RT ANALYSIS WITH THE WEIBULL-GAUSSIAN CONVOLUTION MODEL

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Abstract

The author analyzed the reaction time (RT) distribution using the convolution of Weibull and Gaussian distributions. RT analyses have often been conducted with the ex-Gaussian model, whose hazard function converges to a constant. However, for the visual search task, the hazard is not always stable; there is a possibility that the hazard directs upward. Hence, the author replaced the exponential of the ex-Gaussian with the Weibull and suggested the Weibull-Gaussian model. The Weibull-Gaussian model is a general-purpose model that has backward compatibilities to the ex-Gaussian and Weibull, because each of them is a special case of the Weibull-Gaussian. The author conducted the visual search experiment and then fitted the model and estimated the parameters. The results revealed that both the ex-Gaussian and Weibull were not sufficient for RT distributions, and the author concluded that the flexibility of the Weibull-Gaussian was of benefit for the analysis of RT distribution.

This article has two aims. The first aim is to provide a new distribution function that improves the disadvantages of the ex-Gaussian and Weibull functions. The author calls the new model Weibull-Gaussian, which is a convolution of the Weibull and Gaussian distributions. The second aim is to analyze RTs of a visual search task using the Weibull-Gaussian. In the analyses, the author will highlight qualitative information from the RT distribution.

To analyze RT distributions, the ex-Gaussian and Weibull functions are often used for fitting models (Logan, 1992; Zandt, 2002). However, apart from their merits, each has a demerit. A merit of the ex-Gaussian is its ability to handle noise times such as locomotive time to respond. Handling noise is important for psychology because aside from the main process time, almost all RTs generated by humans contain such noise. A demerit of the ex-Gaussian is its constant hazard function, which is provided by *PDF(t) / (1 – CDF(t))*, where PDF is the provability density function, and CDF the cumulative distribution function. Not all hazard functions of psychological experiments reveal a constant shape. For example, consider a visual search. If an observer searches with the elimination strategy, or checks on an object by object basis. (Figure 1a), the hazard function would direct upward (Peterson, Kramer, Wang, Irwin, & McCarley, 2001). Then, the constant hazard is problematic to RT analysis for psychology. Compared to the ex-Gaussian, the Weibull function can provide a flexible hazard function. However, the Weibull cannot treat noise times. As mentioned earlier, handling noise times is important for psychology. Hence, it can be said that the ex-Gaussian and Weibull each have a problem.

To solve the problems of the Weibull and ex-Gaussian, the author suggests a new distribution, namely, the Weibull-Gaussian model. The Weibull-Gaussian is a convolution of Weibull and Gaussian distributions. The Weibull-Gaussian is derived when the exponential component of the ex-Gaussian is replaced with a Weibull. The PDF of the Weibull-Gaussian is then described as

$$f(t) = \int \frac{\lambda}{\nu}\left(\frac{y}{\nu}\right)^{\lambda-1} e^{-\left(\frac{y}{\nu}\right)} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-y-\mu)^2}{2\sigma^2}} dy\,. \quad (1)$$

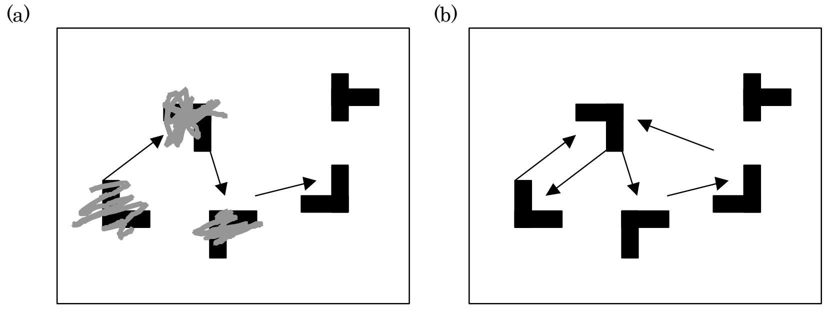

Figure 1. Schematic model of two search types. In the left panel, the observer investigates objects one by one. In the right panel, the observer investigates objects randomly or simultaneously.

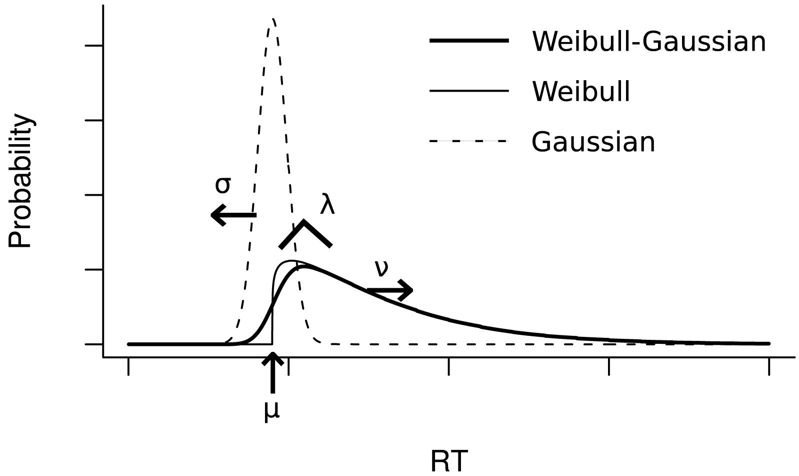

Figure 2. A schematic model of the Weibull-Gaussian distribution. The Weibull-Gaussian (bolder solid line) is a convolution of the Gaussian (broken line) and Weibull (thin line). To know about the parameter, see the main text.

The Weibull-Gaussian has four parameters: μ , σ , ν , and λ . The parameter μ is the mean of the Gaussian component (Figure 2). The μ indicates a base line or the boundary of the faster side of the Weibull component. If the base line, μ, is shifted by training, it can be said that the observer's limit for the task is changed. Parameter σ is the variance of the Gaussian component. In other words, σ reflects the fluctuation size of noise times. When this σ decreases continually, the data approaches to logical values, which have less noise time. The parameter ν reflects the size of the main component. This parameter (ν) can be regarded as a sort of variance*. A large fluctuation of the main process time would show a large ν value; however, sophisticated responses would show a small ν value. The parameter λ is the shape parameter belonging to the Weibull component. This shape parameter reflects the differences of the types of the main cognitive process, and dominates the shape of the hazard function. In visual search tasks, if an observer checks from an object to an object (Figure 1a), the shape parameter would be larger than 1 (Peterson et al., 2001). If an observer checks all objects simultaneously or randomly (Figure 1b), the shape parameter would be converged to 1 (Horowitz & Wolfe, 1998).

The Weibull-Gaussian has backward compatibilities to the ex-Gaussian and the Weibull. When the shape parameter equals to 1, the Weibull-Gaussian becomes identical to the ex-Gaussian. When the noise size decreases continually, the Weibull-Gaussian converges to three-parameter Weibull. Hence, it can be said that the Weibull-Gaussian seamlessly combines the Weibull and ex-Gaussian.

To analyze the RT and to demonstrate the advantages of the Weibull-Gaussian, the author conducts a visual search experiment.

Methods

The author conducted a visual search experiment and then, with the Weibull-Gaussian model, compared the first half and the second half of the session.

Participants

Ten participants were recruited voluntarily.

Apparatus and Stimuli

The experiments were conducted with a custom program running on Windows XP. Responses were collected with a PC keyboard. Experimental stimuli were presented on a 17 in computer display. All search arrays were composed of 12 objects: 11 distractors (L shaped object), and 1 target (T shaped object). The Ls were rotated by 0^o, 90^o, 180^o, or 270^o. The target T was rotated by 90^o or 270^o. A display was divided into an 8 x 6 grid, which was not actually displayed. The objects were then randomly deployed to the 12 cells for each array. The height and width of the objects were 23 x 23 mm on the display. The distance between a participant and the display was approximately 50 cm; however, it was not fixed by any apparatuses.

Procedure

The task was to detect a target among distractors and respond as quickly as possible, while ensuring that responses were accurate. Subjects responded on the basis of the direction of the target by pressing a key: the z or backslash key. A trial was begun with the display of a fixation point for 50 ms, followed by a search array. The search array was displayed until the participant responded. When a response occurred, the search array was replaced with a blank screen in which a short beep was provided as feedback of the correction. The blank screen was displayed for 1000 ms and followed by a fixation of the next trial. One experimental session was composed of 26 blocks, and a block was composed of 24 trials. After each block, there was a compulsory 10 s blank for rest.

Results

For the analysis, all error trials were eliminated from the data. The percentage of correction was 98.50%. All fittings were conducted using the nonlinear least square method on GNU R. The actual calculation of convolution was implemented by the Fast Fourier Transform. The transition of the response times is plotted in Figure 3. The author separated the experimental session into a first (blocks 1–13) and second half (blocks 14–26), and called them the first period and second period, respectively. The arithmetic means were 830 ms for the first, and 722 ms for the second period.

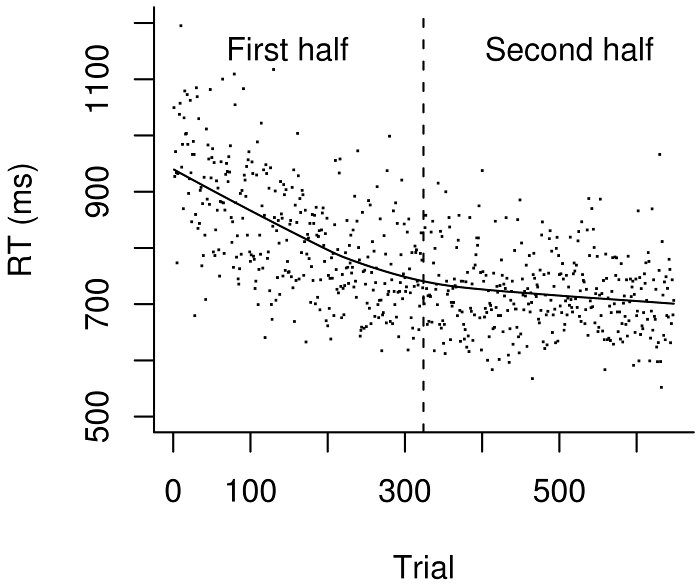

Figure 3. Scatter plot of RT by trial number. The averaged RT of the second half of the session is actually faster than that of the first half. One dot indicates a mean time on a trial over all subjects.

Model Comparison

The goodness of the Weibull-Gaussian, ex-Gaussian, and three-parameters Weibull were compared on the data obtained. The Weibull-Gaussian was given by Equation 1, and the ex-Gaussian by

$$f(t) = \int \frac{1}{\nu} e^{-\frac{y}{\nu}} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-y-\mu)^2}{2\sigma^2}} dy. \quad (2)$$

Further, the Weibull was given by

$$f(t) = \frac{\lambda}{\nu}\left(\frac{y-\mu}{\nu}\right)^{\lambda-1} e^{-\left(\frac{y-\mu}{\nu}\right)^{\lambda}}. \quad (3)$$

Model fittings were conducted for each participant's empirical distribution function, which was expressed as

$$\hat{F}(t) = [\text{ number of observations less than or equal to } t]/n.$$

Note that although the author previously expressed all the functions in accordance with the PDF (Equations 1–3), the actual fittings were conducted with the CDF.

The estimated parameters and goodness of fits were summarized in Table 1. The Weibull-Gaussian shows the smallest value of AIC and χ^2 for both the first and second periods (see Table 1). In other words, the Weibull-Gaussian was the most suitable model among the three.

Analyses of Visual Search

The author compared the RT of the first and second periods using the Weibull-Gaussian. Histograms for each period are shown in Figure 4. As shown in the histograms, the shape of each distribution differed. The largest difference was reflected on ν (the size parameter of the main cognitive process); ν changed from 423.12 to 312.39 ms (t = 8.79, p < .05, Cohen's d = 1.6). The shape parameter also showed significant transition; λ changed from 1.26 to 1.17 (t = 2.76, p < .05, d = 0.62). However, σ and μ did not show such large changes (for σ, t = –0.53, p = 0.6, and d = –0.24; for μ, t = 1.90, p = 0.08, and d = 0.3).

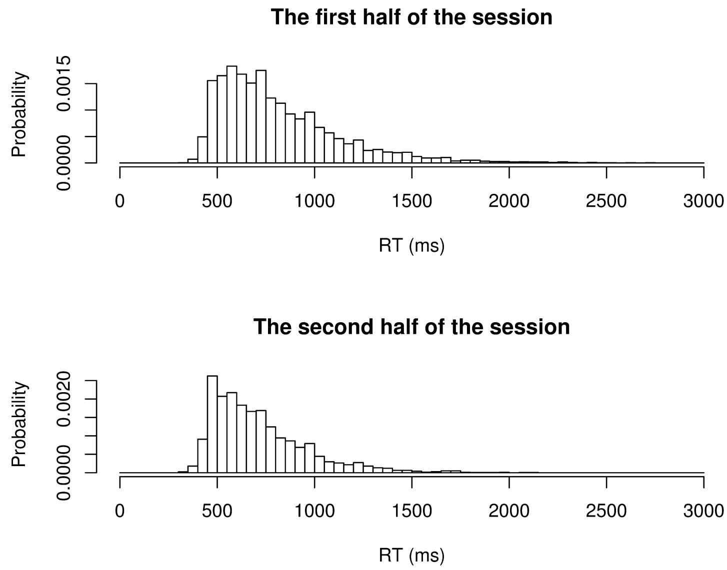

Figure 4. Histograms for each period. The upper panel is the histogram of the first period and the lower panel, that of the second period. The shapes of the distributions differed from each other.

Table 1

Estimated Parameters and Goodness of Fit

Period	Model*	μ	σ	ν	λ	AIC**	χ^2
First	W-G	463.45	20.02	423.12	1.264	–1883.72	0.08
	W	456.61		430.19	1.289	–1871.97	0.11
	E-G	522.75	82.45	345.91		–1717.29	0.22
Second	W-G	446.50	24.04	312.39	1.168	–1889.41	0.09
	W	438.09		322.33	1.208	–1842.93	0.18
	E-G	474.57	55.70	275.14		–1765.62	0.15

* Weibull-Gaussian, Weibull, and ex-Gaussian are abbreviated as W-G, W, and E-G, respectively.

** AIC is Akaike's information criterion.

Discussion

It was revealed that the Weibull-Gaussian model was suitable for the analysis of RT distributions. As shown in the result, noise time certainly exists, although the response of the experiment was collected in a very simple manner. If the responses were collected in a more difficult way—for instance, using a reaching task—the noise component would be larger and dominate the distribution. For this reason, ignoring the noise component must result in worse fitness, as in the case of the simple Weibull. Hence, the noise parameter of the Weibull-Gaussian is necessary.

The shape parameter did not show 1, and it was changed during the sessions. Fixing this parameter to 1 should result in the deterioration of the fitness, as in the case of the ex-Gaussian; this will also result in the loss of important information—primarily, the cognitive type. Hence, adding the shape parameter to the RT model is recommended.

Applying the Weibull-Gaussian revealed which components of RT were changed in task repetition. The most notable difference between the first and second periods was their fluctuation size (ν). As compared to ν, the base lines (μ) did not show such a difference. These facts tell us that most of the speedup in the second period was not caused by the transition of the participants' ability; rather, it was caused by sophistication. In other words, fewer inefficient responses resulted in faster average RT.

The types of the search process also changed from the first to the second period. The shape value was clearly larger than 1 in the first half; however, it approached to 1 in the second half. These facts indicate that an observer searched object by object during the first period and searched the objects simultaneously during the second period. In other words, the participants were able to search a broader area at a time.

As we have seen above, all parameters of the Weibull-Gaussian are important and should not be ignored. Further, the RT distributions did not show a clear shape of ex-Gaussian or Weibull; rather, it showed their eclectic shape. Hence, the Weibull-Gaussian, which contains the ex-Gaussian and Weibull functions, is suitable to analyze RT distributions.

References

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Footnotes

*The actual variance of the Weibull function is expressed as

$$\nu^2\Gamma\left(\frac{\lambda+2}{\lambda}\right)-\nu^2\left[\Gamma\left(\frac{\lambda+1}{\lambda}\right)\right]^2.$$