S.S. STEVENS' LEGACY: AN IDEAL PSYCHOPHYSICAL LAW?

Lawrence M. Ward
Department of Psychology and The Brain Research Centre
The University of British Columbia, 2136 West Mall, Vancouver, B.C. V6T 1Z4 Canada

Abstract

S.S. Stevens left a legacy that still resonates today, although there are some who feel that we should depart from the program he and Fechner before him pursued, and others who have refined that program considerably. In this paper I reiterate some of that legacy, in Stevens' terms, and speculate about how we can create an ideal psychophysical law, similar in derivation and usefulness to the ideal laws of physics, with the ideal gas law as a physical example. I also develop a few of the implications of this ideal law for the apparently diverging traditions of biological and physical approaches to psychophysics.

S.S. Stevens continued and changed in significant ways Fechner's program of conceptual and empirical psychophysics. In his famous papers "On the Psychophysical Law" (Stevens, 1957) and "To Honor Fechner and Repeal His Law" (Stevens, 1961) he made a convincing case that Fechner's Law should be replaced by a power law, now named for him. In spite of more recent modifications and extensions (e.g., Norwich, 1993), that simple and elegant formulation has continued to appear in diverse theories and applications throughout psychology. Moreover, although ironic, his approach has been shown to result in scales for subjective (intensive) quantities that have the same status as classical measurement based on commutative and associative operations (Narens, 2002; cf. Ward, 1991). In this paper I use another of Stevens' fundamental contributions, his extension of the power law to account for context effects, to speculate about a form of ideal psychophysical law (or at least an "ideal" form of the power law), that might perform analogously to ideal laws in physics. This speculation raises several issues about the usefulness of the pursuit of the psychophysical scaling program. Before engaging in these thoughts, however, I need to review some aspects of how such laws function in classical physics (see Feynman, 1967).

Laws in Physics

There are two kinds of laws in classical, macroscopic, physics. One is a definition, an axiom, or a theorem in a theory. For example, $F = ma$ is the definition of the physical concept of force in Newton's theory of mechanics; it is called Newton's Second Law of motion. The other kind of law is an empirical generalization. For example, Ohm's Law, $E = IR$, is a generalization from many experiments relating the voltage (electromotive force, E) in an electrical circuit to the resistance (R) across which the voltage is read and the electrical current (I) flowing through that resistance. Such laws are used by thousands of physicists every day and are absolutely indispensable to physical practice. They perform several functions, such as summarizing empirical results (empirical laws), and serving as statements of physical principle with which new theories must be consistent. They also serve as targets for new physics, which extends or restricts their range of applicability, or replaces them with more general and useful laws. Finally, they are practically useful: they are the foundations of engineering, which takes them as established and applies them in building artifacts.

A nearly "ideal" physical law is the ideal gas law. This law is based on early experiments by physicists such as Robert Boyle, who in 1660 reported that, at constant temperature (*T*), the pressure (*p*) exerted by a gas on its container varied as a function of the volume (*V*) of the container (Boyle's Law), and Joseph Louis Gay-Lussac (preceded by experiments by Jacques A.C. Charles), who in 1802 reported that, at constant pressure, the volume of a gas varies with the its temperature (Charles'/Gay-Lussac's Law). Combining the two earlier laws and some theory, the ideal gas law is

$$pV = NkT, \tag{1}$$

where *k* is Boltzmann's constant (1.38 x 10^{-23} Joules/Kelvin per molecule) and *N* is the total number of molecules of gas (e.g., Feynman, 1963; Sears & Zemansky, 1960). There is only one trouble with the elegant Equation 1, *it isn't correct*! That is, it holds only at *low* pressures, where the electromagnetic forces between the molecules of gas are very weak and pretty much their only energetic interactions are via collisions, and even then it only holds approximately! It is only exactly correct when the forces between gas molecules are zero. So it doesn't apply in the sun, where pressure is high, and it doesn't apply in a nuclear reactor, and it doesn't apply even in a boiler on a steam engine! Moreover, it is really a very large (some would say infinite) *family* of laws stated in summary form. For example, if we divide both sides by *p*, we have $V = NkT/p$, and we see that for every different sufficiently small pressure *p* there is a different way in which *V* changes with *T* and *N*, for example $V = NkT/0.003$, $V = NkT/0.004$, $V = NkT/0.005$, etc. In fact, in order to make it work for all pressures (so even more versions of the law) physicists have defined an "ideal gas" (which doesn't exist) that has no forces at all between the molecules. So what use is such a law?

An ideal gas is a fiction – there is no such thing in nature. So why have a law about a fiction? That makes the law itself fictional. Well, physicists have made a lot of wonderful progress talking about such fictions. In this case, the derivation of the ideal gas law from Boyle's, and Charles'/Gay-Lussac's Laws made physicists think hard about gases, temperatures, and pressures. Physicists like Boltzmann and others came up with theories about heat and work and eventually with the stupendously successful kinetic theory of gases, the laws of classical thermodynamics, an understanding of phase changes of various kinds of elements and compounds, and even the Kelvin scale of temperature. Moreover, an understanding of when the law failed led to generalizations that worked over a wider range of conditions. Now physicists talk, for example, about adiabatic processes (in which no heat enters or leaves the system), isochoric processes (in which the volume remains unchanged), and isobaric processes (in which pressure remains constant) as a matter of course, always being aware that whenever they are dealing with macroscopic processes relating to gases they must consider the reciprocal relationships between pressure, volume, and temperature. Finally, the law is *approximately* correct in many situations, and as long as care is taken about the conditions it can function very well in predicting one of its variables from knowledge of the others.

An Ideal Psychophysical Law?

Let us consider Stevens' Law in the light of what we know about physical laws. Stevens (e.g., 1957, 1961) preferred the form

$$\Psi = a(\Phi - \Phi_0)^m, \tag{2}$$

where Ψ is psychological magnitude, Φ is physical magnitude, Φ_0 is absolute threshold, and *a* and *m* are constants. More simply, with $\Phi_s = \Phi - \Phi_0$,

$$\Psi = a\Phi_s^m. \tag{3}$$

Stevens measured Ψ with direct scaling, and this measurement process has been axiomatized (e.g., Krantz, 1972; Narens, 1996, 2002). [I won't talk any more about measurement per se, except to say that the results of direct measurement are observables and that by making additional assumptions we can consider the observables to establish ratio scales for psychological magnitude; see Narens, 1996, 2002; Ward, 1991.] First, Equation 3 is simple and elegant and summarizes a huge body of data rather well (e.g., Stevens, 1975). Of course, neither *a* nor *m* is really constant; the measure constant *a* varies with many factors (e.g., Borg & Marks, 1983) and the exponent *m* varies in particular across modalities. Unfortunately *m* also varies with many other factors as well (e.g., Poulton, 1989). Moreover, it never exactly describes the results of even an "ideal" experiment, such as one run by Stevens himself. Thus, Stevens' Law is a limited one that only applies approximately under special conditions (just like the ideal gas law). Second, even conceding the variation of *m*, measured under Stevens' conditions *m* is roughly consistent across various scaling methods, in particular between magnitude estimation and production and cross-modality matching. That is, cross-modality matches of loudness to brightness, for example, are roughly linear with each other when the physical stimuli are expressed in energy units, as predicted by the fact that when magnitude estimations of each continuum separately are fitted with Equation 3, both exponents are 0.3. This consistency means that setting a single exponent by convention immediately yields exponents for all other continua that can be scaled in this way. Stevens (e.g., 1975) established a system of scales, including the sone and bril scales (that are still contained in ISO standards), with standard exponents for over a dozen sensory continua. Using constrained scaling, in which subjects are taught and calibrated to a standard scale, and using the sone scale (exponent 0.3 for sound intensity and 0.6 for sound pressure) as the standard scale, we can diminish most of the biases that lead to cross-individual and cross-lab variability in exponents (e.g., West, Ward & Khosla, 2000). If we accepted Stevens' system of exponents by convention, we could establish a uniform system of measuring the psychological magnitude of stimuli from many sensory and non-sensory continua (cf. Krantz, 1972).

It would seem that training people to use an agreed upon set of scales would be useless when everybody knows that results using these scales are valid only under a standard and restricted set of conditions. And this would render any laws based on such scales, i.e., Stevens' Law, similarly restricted in their applicability (as are physical laws). Of course Stevens thought of this problem and proposed one solution to it in his well-known paper on power group transforms (Stevens, 1966). Curiously, the implications of this paper for psychophysics have been neglected, perhaps because Stevens did not develop them far enough. What Stevens (1966) noticed was that when various conditions impacted the processing of sensory/perceptual stimuli, they tended to change the exponent of the power function rather than changing the form of the function itself. In his paper he described a large number of experiments in auditory and visual masking and also generalized the result to auditory recruitment. In all cases, when the target stimulus was less intense than the masker, its apparent magnitude grew as a power function of the stimulus intensity but with an exponent larger than the canonical one for that sensory continuum.

Let us alter Stevens' basic power law equation, in principle, to accommodate the fact that the exponent, *m*, varies across several distinct experimental conditions. Letting α, β, γ, etc. stand for multiplicative factors that summarize the effect of these factors on *m*, as divined by Stevens (1966), we have

$$\Psi = a\Phi_s^{\alpha\beta\gamma m}. \quad (4)$$

We assume that, under certain ranges of relevant variables, α = *f(A)*, β = *g(B)*, γ = *h(C)*, etc. Thus, once we have defined how the canonical exponent changes under certain conditions, we can broaden the applicability of Stevens' Law by complicating it. This would work for any factors *A, B, C* that had consistent and lawful effects on the exponent, as in Stevens (1966).

As an example, consider the data of Pollack (1949) replotted by Stevens (1966; Figure 1). Listeners heard monaurally 3 sec of continuous speech alternated with 3 sec of noise-masked speech at various levels. Listeners adjusted the level of the speech in quiet to match loudness of the speech in noise. From these data Stevens (1966) determined that the ratio of loudness exponent of speech in noise to that in quiet, α, grew as a power function of the noise pressure with an exponent of 0.16. From Figure 6 in Stevens (1966) I determined that the function is $\alpha = \frac{1.23N^{0.16}}{0.6} = 2.05N^{0.16}$ where N is the sound pressure of the masking noise. Thus, for such masked stimuli, the psychophysical power law for loudness, usually expressed in Stevens' terms as $L = aP^{0.6}$, would become

$$L = \begin{Bmatrix} aP^{0.6} & for\ P > N \\ aP^{0.6\alpha} & for\ P \le N \end{Bmatrix}. \tag{5}$$

For example for a masking noise of 70 dB (which is 0.631 dyne/cm^2) it would appear as $L = \begin{Bmatrix} aP^{0.6} & for\ P > 0.631\ dyne/cm^2 \\ aP^{1.9} & for\ P \le 0.631\ dyne/cm^2 \end{Bmatrix}$. Moreover, Equation 5 could be inserted into theories that required loudness as an intervening variable, and proper expressions for theoretical variables dependent on loudness could be derived for various ranges of masking noises. This would make these expressions more complicated, but life is at least as complicated as the nonliving physical world, and the physical description of that is very complicated indeed. We should not shrink from complication when it is warranted.

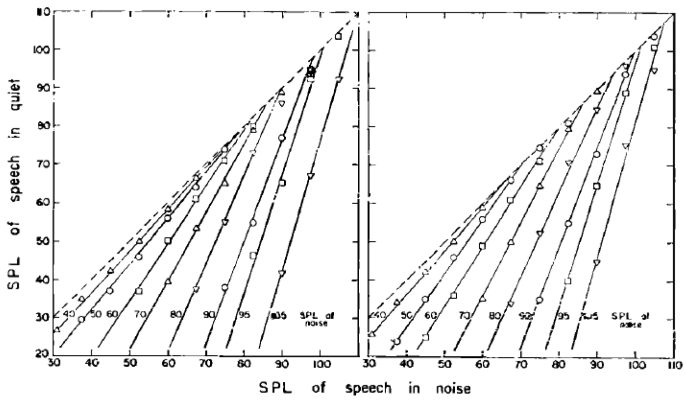

Figure 1. Data on loudness matching from Pollack (1949) replotted by Stevens (1966).

Why Preserve Psychophysical Scaling and the Psychophysical Power Law?

Perhaps the greatest need for psychophysical laws is in practical applications. Some countries, such as Sweden, depend heavily on psychophysicists for advice about environmental and industrial impact on workers and residents. The Borg scale is used in thousands of medical settings to assess stress, pain, and exertion, probably having saved many lives (e.g., 2003). Numerous other example can be provided. One example in my own experience is a very useful model of aircraft-noise-induced annoyance developed by Fidell, Schultz and Green (1988). The model used the percentage of people highly annoyed as its basic variable, based on the work of Schultz, who developed an average curve relating this variable to the amplitude of the noise. The model assumed that noise exposure could be summarized by a variable closely related to the loudness of the noise, and used Stevens' power law for loudness in sones, with an exponent of 0.6 (for the pressure amplitude of the noise) to characterize the noise dose. Without some such assumption the model is impossible and useless, although the particular value of the exponent is not critical to its usefulness; 0.6 was preferred because of the extensive body of psychoacoustic data consistent with this value (e.g., Hellman, 1991). The model assumes that a person's annoyance is an exponential function of the noise dose, and that the person will report high annoyance if the value of annoyance exceeds a criterion (as in signal detection theory). The curve of percent highly annoyed versus noise level produced by this model closely resembles Schultz's empirical curve, but is more useful, as it accords closely with both the intuition that this curve should be sigmoidal, and also fits most of the existing data points over a wide range of noise levels. Moreover, it provides an alternative account of differences between communities in their responses to noise, as arising mainly from differences in the criterion adopted for reporting high annoyance, and not in differences in the basic psychoacoustic response to the noise. I found this idea very useful in a recent court case in which one community had responded grossly differently to a particular level of noise than had another – probably because of political factors inducing them to lower their reporting criterion.

There are also purely scientific reasons for adopting such a law. Hellman (1991) makes a compelling argument for adopting the sone scale as the canonical scale for the loudness of pure tones. She points out that results of matching, loudness additivity, and other accepted psychoacoustic findings severely constrain the relation between loudness and sound intensity to be very close to the power law with exponent 0.6 (for pressure), which is the exponent yielded by the sone scale. Moreover, loudness often enters fundamental theories in physiological acoustics, such as Zwislocki's (e.g., 1994) concerning the relationship between discrimination and loudness. He too assumed a loudness function similar to the sone scale in using loudness as an intervening variable in a theory of intensity discrimination. Indeed, he assumed that subjects discriminate loudnesses, not physical intensities! Similar cases can be made for concepts in other modalities. Thus, it would seem that psychophysical scaling provides convenient measurement of important intervening variables in the study of sensation and perception, and even in cognition (e.g., Algom, 1992).

Physical and Biological Traditions in Psychophysics

For much of its early history, psychophysics followed the form of physics, probably because it was limited to outer psychophysics. Now that inner psychophysics is more possible using neural imaging, and we know that both Fechner and Stevens were probably wrong in their assumptions about inner psychophysics, the physical approach has come into disfavor. Many psychophysicists, including myself, have turned their attention more toward the pursuit of neural and evolutionary explanations for psychological phenomena. Moreover, it has been argued that the heavy dependence of psychological magnitude on context, partly arising from difficulties of abstracting attributes from objects, pose insurmountable problems for the physical approach of trying to discover useful scales to measure psychological magnitude, and discovering simple algebraic laws relating fundamental variables such as loudness measured on these scales (Lockhead, 1992). The two approaches seem complementary to me – neither promises complete understanding and both have significant difficulties to overcome. To eschew simple physics-type algebraic laws is to give up one of our most useful tools.

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