AN EXTENSION OF SYSTEMS FACTORIAL TECHNOLOGY (SFT) TO ARBITRARY NUMBERS OF SUB-PROCESSES

Haiyuan Yang[a], Mario Fific[b] and James T. Townsend[a]*
[a] *Department of Psychological and Brain Sciences, Indiana University, Bloomington, IN, 47405,USA*
[b]*Max Planck Institute for Human Development, Center for Adaptive Behavior and Cognition, Berlin, 14195, Germany*
**jtownsen@indiana.edu*

Abstract

The present study began by exploring the precise behavior of the serial exhaustive Survivor Interaction Contrast (SIC) function for n=2. We found that: A. there must be an odd number of crossings for any distributions. B. a rather mild condition known as log-concave, is sufficient as a guarantor of a single zero crossing. The second major part of the study explored how the SIC signatures act when the number of sub-processes (n) is varied: We provide a generalization of the SIC function to arbitrary number of sub-processes, as well as a theoretical analysis of the SIC in its generalized form for both parallel and serial models in conjunction with both the minimum time and maximum time stopping rules. Based on rigorous proofs, we show that even in the multi-processes case, SFT is a valid tool in distinguishing mental architectures.

The question of whether people can perform multiple perceptual or mental operations simultaneously (parallel processing) has intrigued psychologists since the late 19th century. Reaction Time (RT) has been the primary measure on this question for a long time (e.g., Donders, 1869; Sternberg, 1966, 1975). It is rather startling that antipodally distinct architectures, such as serial vs. parallel structures, do not necessarily make different predictions in the experimental milieu (Townsend, 1972, 1974). Therefore, it may be necessary to craft experimental designs that maximize opportunities for distinguishing among such structures or mechanisms (Townsend, Yang & Burns, in press).

Our focus here lies within the general approach referred to as *Systems Factorial Technology* (hereafter SFT; see Townsend & Nozawa, 1995; Townsend, 1984, 1992). A number of investigators have made essential contributions to this literature including Schweickert and Dzhafarov and colleagues (Dzhafarov, 1997; Schweickert, Giorgini & Dzhafarov, 2000; Schweickert, 1978; 1982; Schweickert & Giorgini, 1999). SFT relies heavily on mathematical propositions indicating experimental conditions where strong tests of architectures may be found, although other testable features, such as capacity, are also encompassed presently.

The classical RT factorial methodologies were based on mean RTs (e.g., Sternberg, 1969; Schweickert, 1978; 1982; Dzhafarov & Schweickert, 1995; Schweickert & Townsend, 1989). The general theory has subsequently been expanded to architectural identification assays which rely on the entire RT distributions (Townsend & Nozawa, 1995; Townsend & Wenger, 2004; Schweickert, Giorgini & Dzhafarov, 2000). The latter methods encompass diagnostics that test among certain architectures and other process aspects that are invisible under the standard mean RT approaches.

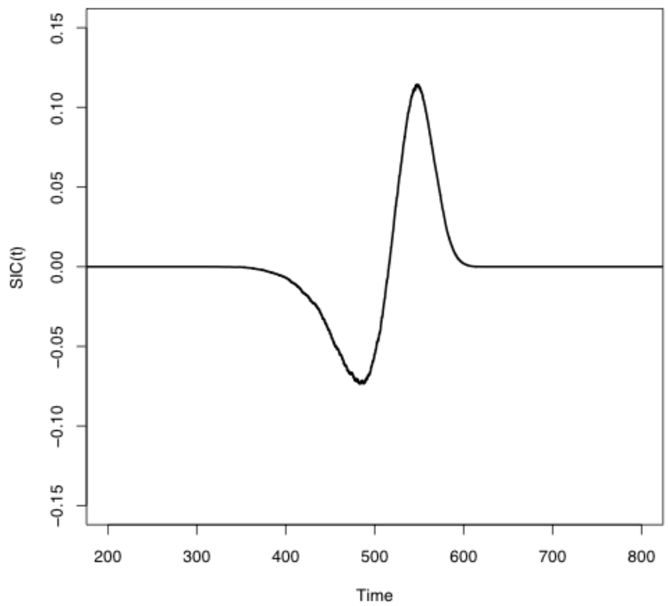

Fig. 1. Predicted survivor interaction contrast form for Serial-AND processing models.

Despite the successful deployment of this technology to date, there are some vital gaps in our knowledge, restricting the applications to a limited class of models and data sets.

Theory

We know that , for n=2, serial minimum time models predict perfectly flat signatures whereas serial maximum time (i.e., the classical exhaustive processing time stopping rule; see Sternberg, 1969) predictions must include at least one wiggle below and above 0 (see Figure 1). However, although simulations have intimated that there is a single wiggle passing through 0, in a negative-to-positive direction as exhibited in Figure 1, this has not been mathematically proven to be true for any specified class of distributions. The following theorem has been proven:

We first proved that exhaustive serial processing inevitably predicts an odd number of 0-crossings in the n=2 case. This result appears in Proposition 1:

Proposition 1: Let $n = 2$ *. Under the selective influence condition of one-point crossing, the exhaustive serial SIC must have an odd number of non- trivial crossovers with horizontal axis in the interval* $(0,+\infty)$.

Next, we showed that a certain readily-met mathematical stipulation (log concavity, basically curves do not increase too rapidly) results in a rather benign condition which is sufficient to ensure a single 0-crossing for n=2, again, in the case of exhaustive serial processing:

Proposition 2. Under the same assumptions as with Proposition 1, the $SIC(t)$ *crosses the time axis only once in the interval* $(0,\infty)$*, if either* $F_{XH}(t) - F_{XL}(t)$ *or* $F_{YH}(t) - F_{YL}(t)$ *is log concave.*

A serious theoretical lacuna has been that the behavior of our RT signature is unknown for n>2, in the case of all studied serial and parallel processes. We thus desire to investigate and determine the quite intriguing behaviors in the case of the serial and parallel models with varying stopping rules, and for arbitrary values of n. We were able to attain powerful and general results for all cases except the exhaustive serial processing, when n>2. The other main propositions will be put in italics as above. However, because the conclusions regarding serial exhaustive processing for n>1, is more speculative that 'tentative proposition' is put in ordinary font.

The first result for n>2 concerns minimum time parallel processing (the so-called class of 'race models'):

Proposition 3. *In the presence of selective influence and independence, minimum time parallel processing predicts that the SIC curve will always be positive as a function of time t, for every n≥2.*

What about parallel processing when "all the horses must finish the race", that is, exhaustive parallel processing? The answer is found in the next proposition:

Proposition 4. *Assuming selective influence and independence, parallel exhaustive process models predict underadditivity in the survivor function when an even number of channels are processed, yet they predict overadditivity when an odd number of channels are processed.*

The reader may observe the rather fascinating prediction that whereas the minimum time stopping rule with parallel processing predicts the SIC functions to be positive, whatever the value of n, the maximum time stopping rule predicts a flip-flop of the sign of the SIC functions, being negative when n = 2, 4, or any even number and positive for n = 1, 3, 5,or any odd number!

Serial processing with a minimum time rule makes a very strong and constant prediction for varying n, as shown in the next proposition:

Proposition 5. Assuming *selective influence, the serial minimum time RT survivor interaction contrast is predicted to be zero for all values of n.*

The final system has been the toughest to solve up to now, at least, namely serial processing with an exhaustive (maximum time) stopping rule. Nevertheless, we were able to prove Proposition 6A,B concerning behavior of these systems:

Proposition 6.

A. For any independent n-channel serial exhaustive processing, the SIC function must be negative for small times if n is even, and it must be positive for small times if n is odd.

B. For any independent n- stage exhaustive serial processing, the integral of the $SIC_{ser.AND}^{n}(t)$ function on the whole space is always equal to zero.

Thus, the appearance of the SIC function is determined for small and large time values, since a corollary of Proposition 6B is that if the SIC function starts out negative, it ends being positive and vice versa.

However, we do not yet possess the ability (and maybe lack sufficient mathematical conditions), to prove a natural extension of Proposition 1, namely that for arbitrary n $\geq$ 2, there will always be n-1 0-crossings of the SIC function. Thus, we express this as a potential and still tentative result:

<u>Tentative Proposition 7.</u> Suppose there are n channels to be processed. Then, if for every channel i, the differences, $F_{iH}(t_i) - F_{iL}(t_i)$ are log-concave, then $SIC^{n}_{ser.AND}(t)$ must have $n-1$ zero crossings in $(0, \infty)$.

Figures 2 illustrate simulations of the parallel and serial systems for varying n and the two stopping rules, minimum vs. maximum stopping time.

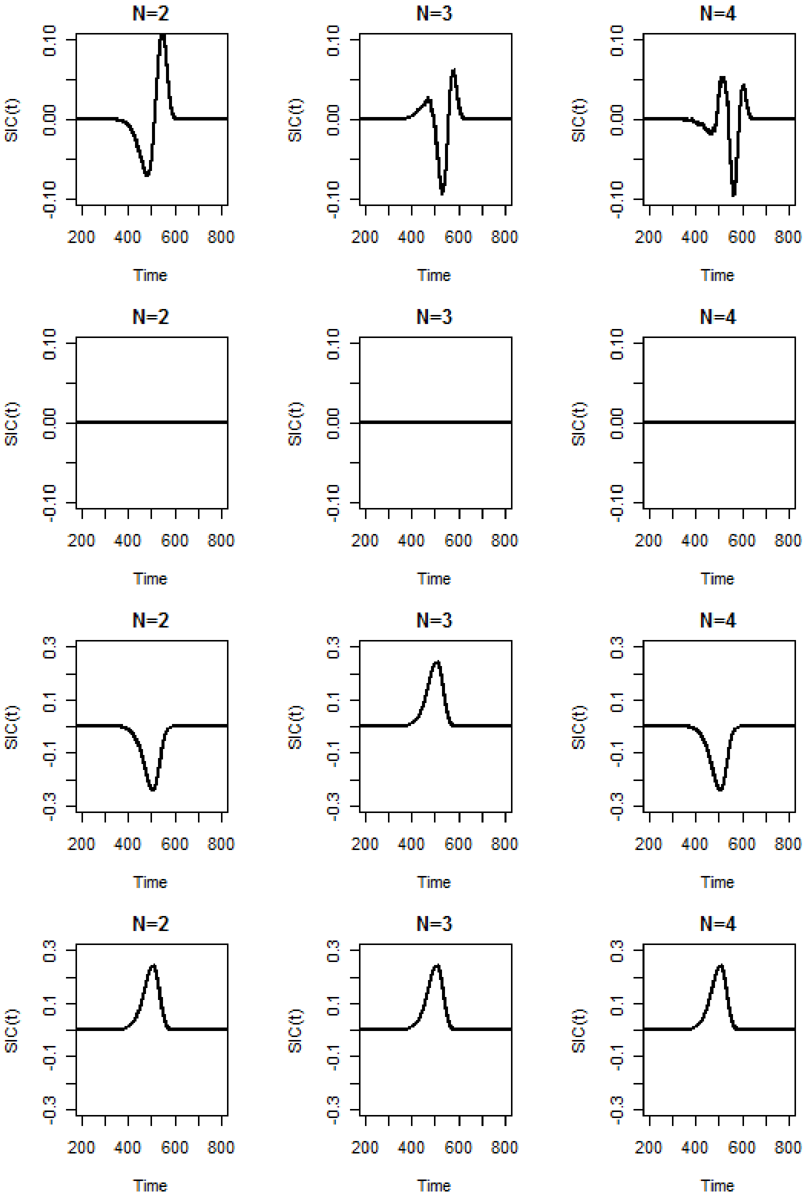

Fig. 2. Predicted survivor interaction contrast forms for Serial-AND (first row), Serial-OR (second row), Parallel-AND (third row) and Parallel-OR (fourth row) processing models, for varying n from 2 to 4.

We continue to seek conditions and the logic which might authenticate tentative Proposition 7 in a general fashion. Nonethehess, it is clear that serial and parallel models make extremely unique and divergent predictions as the number of processes, n, varies. These predictions must now be tested in experimentation.

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