TWO-DIGIT NUMBERS: HOW STRONG IS THE GLUE BINDING THEIR DIGITS?

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Abstract

Are multi-digit numbers represented as a single unique stimulus or decomposed into their constituent digits in tasks of numerical cognition? If the latter, then selective attention should be possible to each of the component digits. We found that selective attention was good for both the decade and unit components of two-digit numbers, but that the latter components were processed in an integral fashion under specific contexts.

The developmental trajectory of two-digit numbers differs from that of other complex stimuli. At early age, the components of complex stimuli tend to coalesce into a "perceptual blob," so that young children experience difficulty at disentangling the constituent dimensions perceptually. The perception is said to be holistic, the dimensions converging in a *Ganzheit*. In the language of Garnerian theory (Garner, 1974; Melara & Algom, 2003), the stimulus dimensions are *integral* dimensions early in development. At a later stage of cognitive development, children become increasingly analytical, so that they are able to decompose the stimulus into its various attributes with relative ease. In Garnerian language, the same dimensions change into *separable* dimensions. With two-digit numbers, this developmental sequence is reversed. Children first conceive such stimuli in an analytical fashion, so that the component digits comprise separable dimensions. At a later stage, children and adults are encouraged to treat the same components as integral dimensions. The unique trajectory bequests adults with a certain indeterminacy in perceiving two-digit numbers. Are two-digit numbers processed in a holistic manner or decomposed into their constituent digits in tasks of numerical judgments? This question formed the topic of the present investigation.

At the outset, when teaching number concepts to elementary school children, two-digit numbers are introduced in an analytical fashion. The extension from single-digits to two-digit numbers along with the place-value principle mandate that the component digits form separable dimensions. Analytic perception of the components is indeed indispensable when learning two-digit numbers. The two digits that make up the number 22 are identical yet refer to different orders of magnitude. It is for this reason that teachers spend considerable time at conveying their students the difference between the numbers 27 and 72. However, with increased experience, children and adults are encouraged to represent those same two-digit numbers in a holistic fashion as a single integral stimulus. Language seems to be the juggernaut molding this development. Language assigns special names to 72 (seventy-two) and 27 (twenty-seven), despite the common components. The language of numbers helps to glue 7 and 2 into the single and unique representation of 72. How strong is this perceptual glue?

Research to date has been equivocal. Consider the *distance effect* (Moyer & Landauer, 1967): The time to decide which of a pair of numerals is larger is inversely related to the numerical distance separating the numbers. Thus, people are faster to say that 7 is larger than 2 than to say that 7 is faster than 6. Is there a distance effect in comparing two-digit numbers? Some studies (e.g., Dehaene, Dopoux, & Mehler, 1990; Fitousi & Algom, 2006) report that there is: People found it easier to say that 68 was larger than 55 than to say that 61 was larger 55, despite the fact that the common teens digit sufficed to decide the comparison (all numbers in the 60 decade are larger than 55). This result supports holistic processing of two-digit numbers. In a similar vein, Brysbaert (1995) produced data that suggest unique representations for each individual number in the 1-99 range. He found that viewing durations for visually presented numerals were a logarithmic function of their magnitude. Brysbaert concluded that, "at least for the integers from 1 to 99, the representation is [a] number line on which each integer has its unique representation" (p. 451).

Other results disagree with holistic processing of two-digit numbers. In another experiment by Brysbaert (1995), a distance effect was found *only* when the two-digit numbers were confined to the same decade. For numbers from different decades, the differing teens terms rendered the unit numbers gratuitous. This result betrays the presence of separable processing of the components digits. In a study by Nuerk, Weger, and Willmes, (2001), the pair 42-58 was compatible because 4<5 and 2<8, whereas the pair 29-63 was incompatible because 2<6 but 9>3. Decision time decreased and accuracy improved for compatible pairs even when the numerical distance was held constant. The presence of such compatibility effects implies lexical and combinatorial operations, i.e., separate processing of teens and units.

Recent research by Fias, Reynvoet and Brysbaert (2001), Damian (2004), and Ischebeck (2003) suggests that single (Arabic) digits are probably processed in a holistic fashion much like pictures. Each individual digit is associated with a unique representation and this representation is typically accessed by a semantic route the way that pictures are. It is moot whether two-digit numbers are associated each with a unique representation. Damian (2004) surmised that, "the … similarity between digits and pictorial stimuli probably breaks down outside the realm of single-digit numbers" (p. 170).

Therefore, it is still unclear whether perception of two-digit numbers occurs in a unified or an independent fashion. Virtually all studies in the literature approached the question from the holistic end. Participants have been asked to compare, name, or classify two-digit numbers as integral mathematical units. The goal was to find out the extent to which the integrality of the two-digit numbers is compromised by combinatorial operations performed on the component (decade and unit) digits. In the present study, we examined the perception of two-digit numbers from the other, composite end. We asked people to respond selectively to a one component of the two-digit number while ignoring the other component. If the glue binding the digits is strong, then selective attention to each digit is impossible. Indeed, selectivity will be compromised to the degree that bonds the component digits of the two-digit number together. Again, how strong is this glue?

In order to find out, we applied Garner's speeded classification paradigm (Garner, 1974) to each component digit of two-digit numbers. A two-digit number was presented on each trial, and the participant's task was to decide whether the teens digit was larger or smaller than 5 (in a complementary task, the digit to respond was the unit digit). The selectivity of attention to the decade digit was tested in the following three conditions. In the *baseline condition*, the participant classified the magnitude of the decade digit with the irrelevant unit digit held at small or large value throughout the block of trials. For instance, the unit digit was always larger than 5. In the *filtering condition*, both the decade and the unit digits varied from trial-to-trial in an orthogonal fashion, yet the participant's task remained that of classifying the former. In the *correlation condition*, the irrelevant digit of units also varied, once in agreement (*positive correlation*: both digits larger or both digits smaller than 5) and once but in conflict (*negative correlation*: one digit larger, the other smaller than 5).

The ability to attend selectively was measured by comparing performance in the baseline condition, in which the irrelevant dimension was limited to a small range of values, with performance in the filtering condition in which the two dimensions varied in an orthogonal fashion. Performance deficit in the latter, *Garner interference* (Pomerantz, 1986), documents the failure of selective attention. Another measure, Stroop congruity, was given by the difference between the two correlation conditions (all stimuli were congruent in the positive block and incongruent in the negative block).

Dimensions that produce significant amounts of Garner interference (and redundancy gains under correlated dimensions) are called *integral dimensions*. Dimensions that do not produce appreciable amounts of Garner interference are termed *separable dimensions*. Are the decade and unit digits of a two-digit number separable or integral dimensions?

Method

Participants: Twelve young men and women, volunteers from the Tel-Aviv University community took part in the experiment in partial fulfillment of course requirement.

Stimuli and Apparatus: All two-digit numbers in the range 11- 99 were used (except for the digit 5). With the unit digit the relevant stimulus, the participants' task was to decide whether this digit was smaller or larger than 5. Participants performed in 5 blocks of trials: in a baseline block in which the magnitude of the irrelevant decade digit was limited to small numbers (1-4); in a baseline block in which magnitude of the decade digit was limited to large numbers (6-9); in a positive correlation block in which units and decades were both either smaller or larger than 5; in a negative correlation block in which units and decades were always of conflicting magnitude; and in a filtering block in which the units and decades varied in orthogonal fashion. In another sequence of 5 blocks, the participants performed in the same tasks with the decade digit as the relevant stimulus. The 5 blocks for one dimension were presented as a unit before presenting the 5 blocks for the other dimension. Within dimension, the order of blocks was random and different for each participants with the constraint that the baseline and correlated blocks were performed successively (but breaks always separated successive blocks).

Procedure: The participants were tested individually. They performed manually, pressing one of a pair of lateralized keys to indicate the magnitude of the target digit. The participants were asked to respond to that digit and ignore the other digit. Reaction times (RT) and errors were recorded.

Results and Discussion

Figures 1 and 2 give the results -- mean RTs for correct responses in the 5 blocks. Figure 1 presents the results for the *decades* digit as the target stimulus, and Figure 2 shows the parallel results for the *units* digit as the target stimulus.

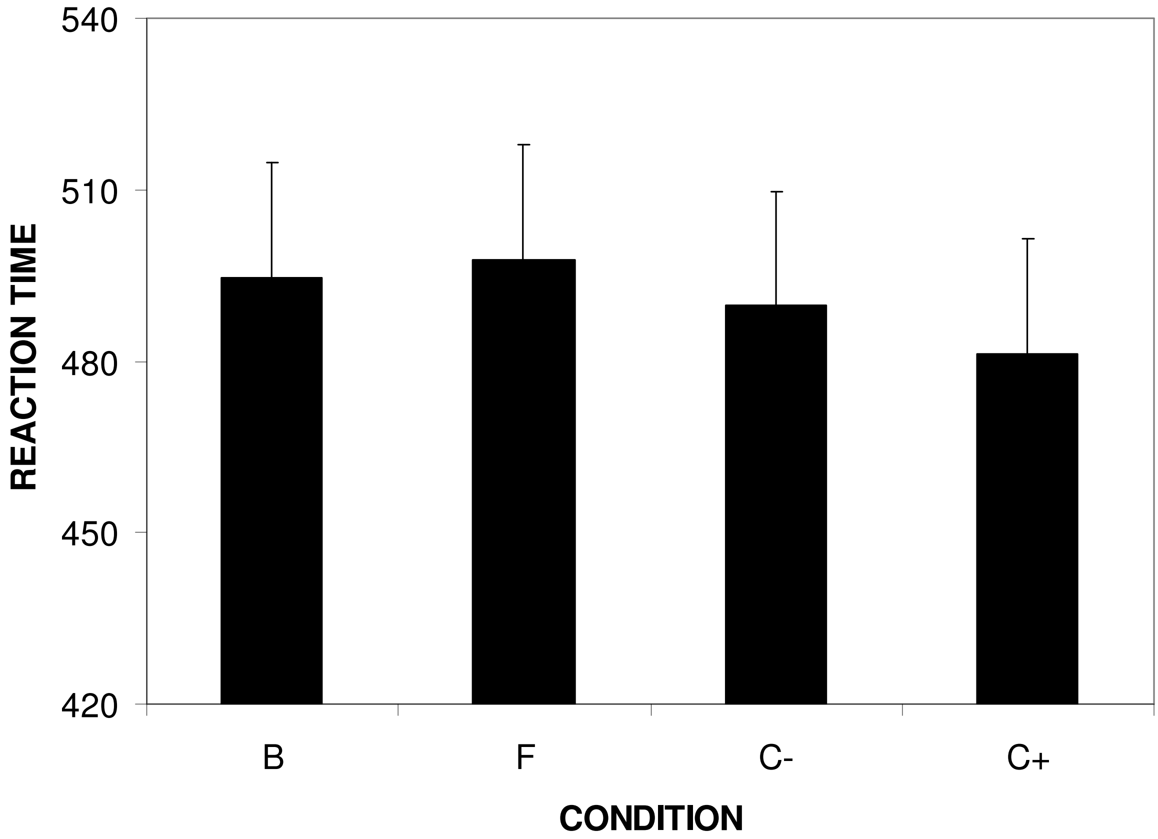

Fig1: Mean RT (in milliseconds) to decide whether the decades digit of two-digit numbers was larger or smaller than 5, in 5 conditions. B: Baseline, F: Filtering, C-: Negative correlation, C+: Positive correlation.

Inspection of Figure 1 reveals that performance was approximately equal under the five conditions. Garner interference was absent: Our participants decided the magnitude of the decades digit in a condition in which the units digit varied in a random fashion as fast as they did in a condition in which the irrelevant units digit was limited to a small window of values. The Stroop effect was also absent: Our participants did not reap redundancy gain when the irrelevant units digit varied in a corresponding fashion with the target decades digit (positive correlation), not did they suffer redundancy loss under negative correlation. Therefore, selective attention to the decades digit of a two-digit number is perfect. Looked at from the decades end of the two-digit number, its processing is entirely separable.

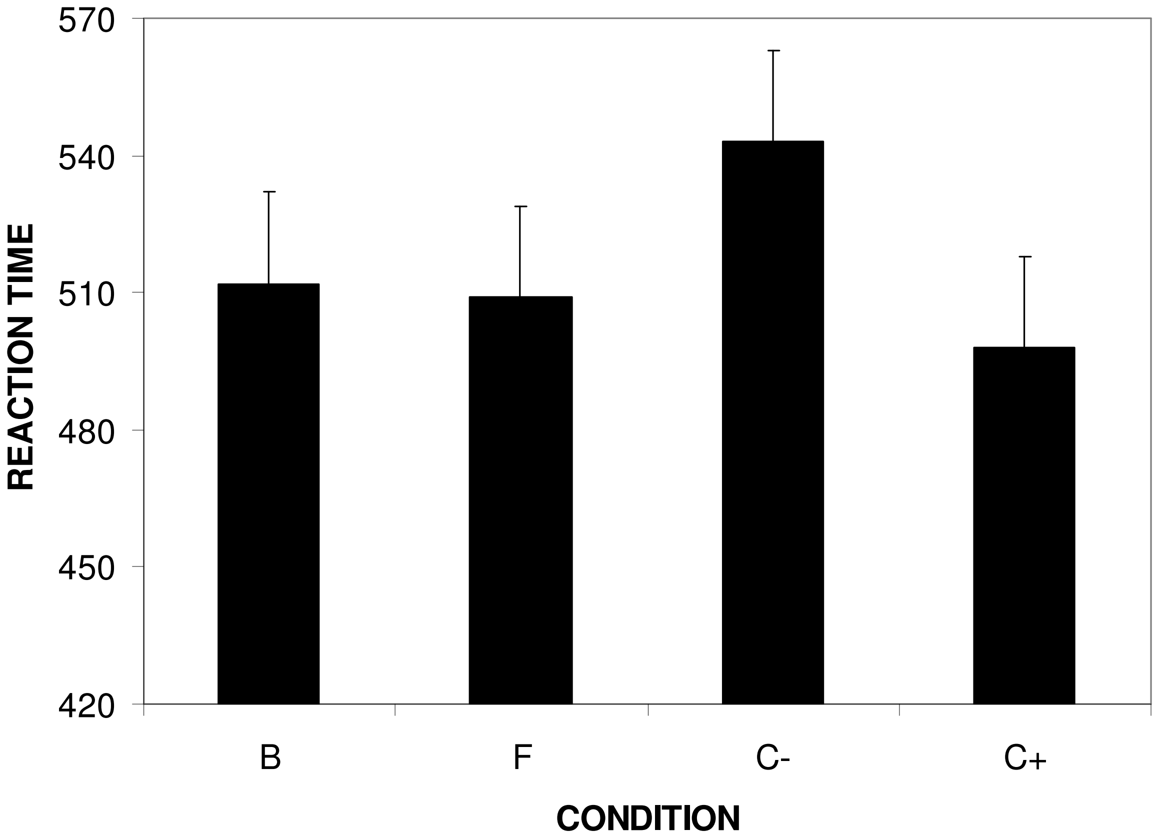

Fig 2: Mean RT (in milliseconds) to decide whether the units digit of two-digit numbers was larger or smaller than 5, in 5 conditions. B: Baseline, F: Filtering, C-: Negative correlation, C+: Positive correlation.

Inspection of Figure 2 reveals a different pattern. Although Garner interference was absent for the units as well, but an appreciable difference between the correlation conditions emerged. When judging the magnitude of the units digit, the participants reaped gain from a matching (yet irrelevant) decades digit, and suffered interference from a mismatching decades digit. Therefore, the processing of the units component of a two-digit number is partially integral. Under random variation contexts -- when the component digits vary in an independent fashion -- people are able to decompose the number into its constituent digits and treat the units digit in a separable fashion. However, under correlated contexts, the digit standing for the units *is* dependent on that standing for the decades. Under such contexts, people are unable to attend selectively to the units component, and they process the decades component ineluctably. Moreover, it is not merely the variation of the irrelevant decade that is registered. That component is processed to the semantic level, otherwise there would not have been a difference between positive and negative correlation. A correlation is present under both of these contexts, yet one context induces a gain (positive correlation), the other a loss (negative correlation) to performance.

Conclusion

How are two-digit numbers processed? This study shows that they are processed largely -- but not fully -- in a separable fashion. Violations of separability appear only for the units, not the decades. Integral processing appears under contexts in which the component digits are predictive of one another. In these contexts, the units components cannot be perceived independently and are affected by the magnitude of the decades. This decade-unit asymmetry is an important feature of two-digit number processing

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