WAVE THEORY: A (NON-RANDOM) WALK THROUGH ESSENTIALS. EXPANSIONS, AND OPEN ISSUES.

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Abstract

Theoretical ideas leading to the *Wave Theory of Difference and Similarity* are discussed. The Poisson distribution of neural events is illustrated and new applications to facial beauty are shown to follow predictions from the theory of ideals.

Let me begin with an observation by Karl Gauss (1821) taken from his famous application of probability theory to errors in measurement:

> Certain causes of error are such that their effect on any one observation depends on varying circumstances that seem to have no essential connection with the observation itself. Errors arising in this way are called irregular or random, and they are no more subject to calculation than the circumstances on which they depend. Such errors come from the imperfections or our senses and random external causes, as when shimmering air disturbs our fine vision. (Trans G. W. Stewart)

Some 30 years later Gustav Fechner applied this idea of Gauss to the measurement of error in our sensory systems. Fechner developed what today is called Ideal Observer Theory – the form of Signal Detection Theory where the decision criterion is midway between the means of two Gaussian distributions – in order to determine the variance of signals within the nervous system. His truly astonishing mathematical model explained the cause for errors in sensory judgments and created a platform for the development of Experimental Psychology (Elemente der Psychophysik, 1860).

Fechner's approach was not without criticism. In spite of the truly phenomenal theoretical advance – an advance by a single theory that drew together previously unexplained and disparate phenomena – an important question remained unanswered. If Gaussian error is a cause for variability in the nervous system, where does this variability come from? As S. S. Stevens argued many years later, Thurstone and Fechner both attempted to create measurements out of chaos and variability.

Link's (1992) Wave Theory adopts the proposal by Rashevsky (1960), McGill (1967) and others that the variability in the nervous system arises from the electrical basis for neural firings. Not from Gaussian error attributable to nothing. No, the very nature of the process of generating the neural event itself produces variability. The process generates Poisson distributed numbers of neural events within a time epoch.

In the case of Poisson processes the mean number of events per epoch and the variance of the number of events per epoch are equal. Furthermore, numbers of neural events have associated probabilities. There are no probability densities having to do with infinitesimally small numbers of neural events, as would be required by a Gaussian assumption. No, there are discrete events and these have associated probabilities. As shown in Figure 1 the various Poisson probability distributions appear to approach a nice symmetry as the average number of events increases. But, these are not Gaussian distributions and the attempt to suddenly argue that they may be represented by such distributions reveals an important misunderstanding of the difference between probability and probability density – a failure still found in some psychological statistics books.

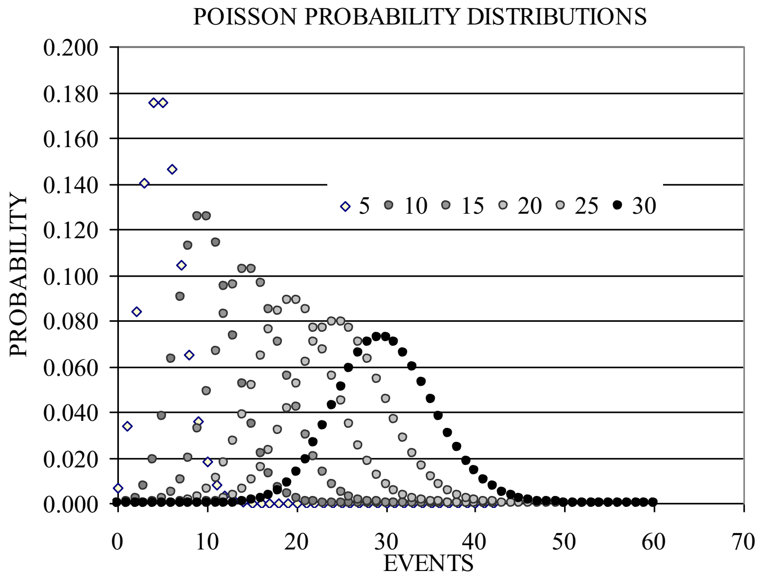

Figure 1. Poisson probability distributions for mean numbers of events from 5 to 30 in steps of five. Symmetry of the Poisson becomes a feature for higher numbers of events.

In Wave Theory these Poisson distributed events occur in the neural system within time epochs. Time epochs are needed because the neural system must accumulate such events before transmitting a signal to neural structures downstream. The minimum duration of such epochs is on the order of 4.5 milliseconds – approximately the time taken for a neural signal to cross the corpus collossum. A consequence of the Poisson distribution is that a neural signal, when plotted as a function of epochs, shows great variability without the requirement of Gaussian noise.

Figure 2 shows examples of 10 such stochastic paths generated from samples of the probability distribution shown in Figure 1 with mean 10. The variability of individual stochastic paths is substantial as shown by one path with the individual values connected by a black line. The mean values within each epoch are shown by the black circles connected by a bold black line.

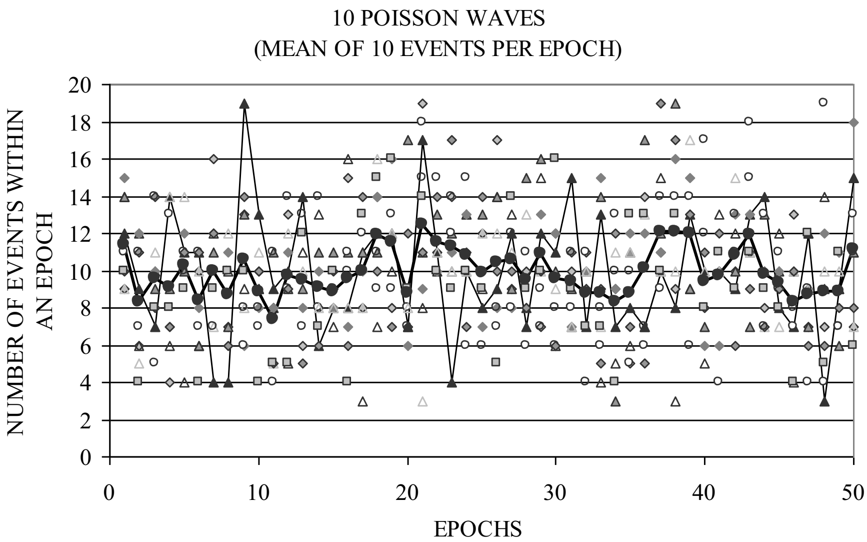

Figure 2. Within each of 50 epochs are the outcomes from sampling 10 Poisson distributions each with a mean of 10.

These Poisson based events may be thought of as arising from different sources but converging on an accumulator of the events. Within each epoch ten different sources are accumulated. Because these are Poisson distributed events the sum is also Poisson distributed. At this point the source of the individual events is lost in the total, like different sources of water converging to form a wave. Figure 3 illustrates the result of summing the number of events in each epoch. Although the individual sources of stimulation are lost the total remains quite variable as is necessary if the mean value and variance are to be equal. There is no Central Limit here that drives the process toward a mean value. There is also no mean value within an epoch, only the accumulation from the individual sources.

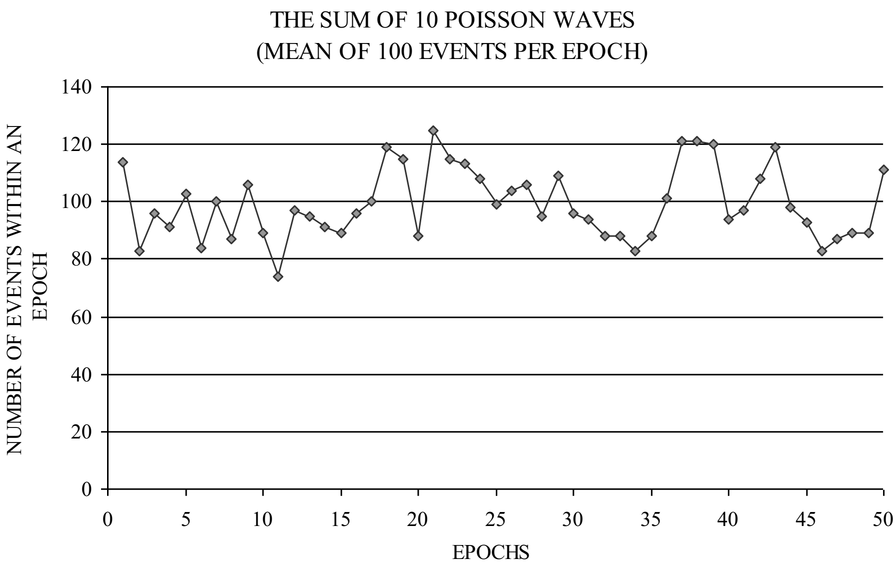

Figure 3. The accumulation of stochastic paths within an epoch produces a Poisson variable with mean 100 and variance 100 with plenty of variability.

A major difference between Wave Theory and other theories of choice and decision making is the nature of the process giving rise to a response. In the case of only two alternative choices the decision process is cast as a sequential accumulation of differences between neural events. The accumulation of differences continues until either of two response alternatives is exceeded. Then the decision process stops and the actual response process begins.

Link and Heath (1975) modeled this accumulation as a random walk based on the ideas originally created by Abraham Wald in formulating the Sequential Probability Ratio Test (SPRT). What distinguished the early work by Link (1975) and Link and Heath (1975) from previous models was the prediction concerning correct and error responses. Previous theories such as SPRT predicted that correct and error response times made by the same response must be equal. Experiments by Laming (1962) and Link and Tindall (1971) showed this prediction to be false, mean correct and error response times differed. Link discovered an assumption in the previous random walk models that could be replaced and then predict differences between correct and error time conditioned on the response made. In the subsequent literature some papers misinterpreted the stricture that the equal response times must be equal for the same response and tested for differences between correct and error times conditioned on the stimulus presented.

Link (2008) reanalyzed data from Laming (1962) for a two-choice experiment where the stimuli were two vertical white stripes. The experimental manipulation was to change, in different blocks of 200 trials, the presentation probabilities for stimuli S_A and S_B. As shown in Figure 4 these probabilities varied from 0.25 to 0.750 in steps of 0.125. Subjects responded R_A or R_B to indicate the occurrence of S_A or S_B respectively. Figure 4 shows that the changes in presentation probabilities resulted in striking differences between mean correct and error times. Bear in mind that the parallel lines are for the same response made either correctly or in error.

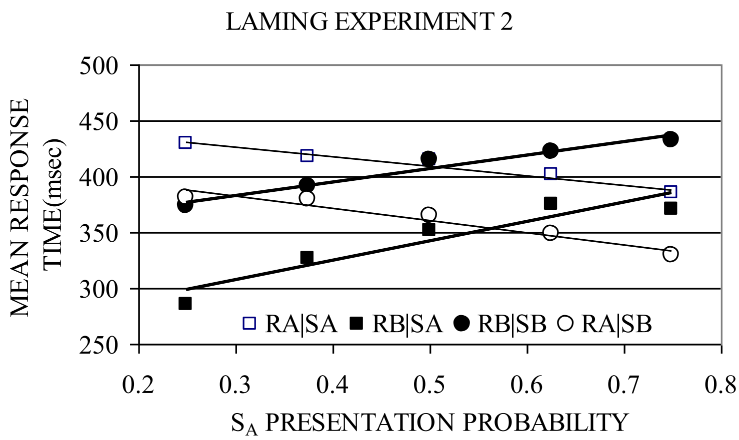

Figure 4. Reanalysis of data from Laming's (1962) two-choice recognition experiment showing quite convincingly differences between mean correct and error times.

What distinguished Wave Theory from the Relative Judgment Theory of Link (1975) and Link and Heath (1975) is the adoption of the Poisson distribution as the basis for neural events. This was not just a happenstance but the result of theoretical investigations showing that this assumption resulted in testable predictions not otherwise available to either Wave Theory or to any other theory of choice or decision. These predictions generated and/or accounted for experimental and theoretical findings of Weber, Fechner, Thurstone, Stevens, Blackwell, Peterson, Birdsall and Fox, Luce, Atkinson and others. As an example Link (1992) derived a relation between values for Weber's Law and Stevens' Power Law exponents showing that Stevens exponents are the reciprocals of Weber fractions. Predictions concerning response proportions, response times and response confidence supported the Poisson assumption.

Testing the underlying assumptions in any theory is always preferable to curve fitting. Ideally one should reduce the number of unknown parameter values to zero so that the outcome of the experiment may be predicted in advance. Simply fitting data with an arbitrary number of parameters, no matter how "efficient," is not a test of a theory but a test of statistical procedures. In this regard the important contribution by Nihm (1984) is not to be overlooked.

During the nearly twenty years since the publication of *The wave theory of difference and similarity* a number of extensions of the basic theoretical concepts developed new applications and theoretical tests. Many are presented in books and contributions to the Fechner Day publications of this Society. Several are of particular importance in the history of psychophysics and decision making. In particular the application of paired comparisons to cross modality matching, the theory of numerical comparisons, the theory of ideals, theory of color discrimination, and predictions concerning response confidence. Let me illustrate one of these, the theory of ideals.

Clyde Coombs in the theory of data provided compelling evidence that comparisons supposed to be between two stimuli are often of a different nature. Two stimuli may be presented for judgment, but they may not be compared against each other. Rather, each may be compared to an ideal, the stimulus nearer the ideal being the one chosen or preferred. The Wave Theory analysis of this type of comparison suggested that a quadratic relation exists between the underlying parameter giving rise to the choice, what is called θA in Link's theory, and physical changes in stimuli to be judged.

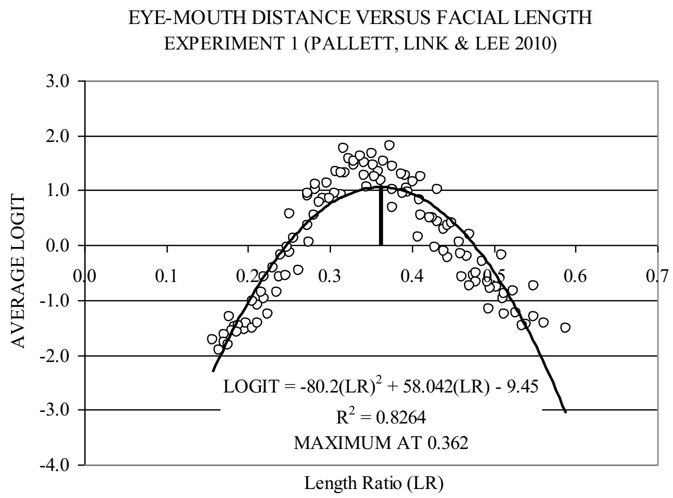

Figure 5. The predicted quadratic relation between the length of face and distance between the eyes and mouth allows for a determination of a "golden ratio" for female facial attractiveness.

In a recent study of preferences for facial attractiveness Pallett, Link and Lee (2010) applied the theory of ideals from Link (1992) to comparisons of female faces. Pairs of faces were shown together and the experimental subject chose the face appearing most attractive. Of course, attractiveness is an individual matter and the most attractive face an individual ideal. Based on the theory of paired comparisons for ideals (Link, 1992) the paired comparison results from each of four experiments were used to compute the "ideal" ratios for the distance between eyes versus the width of the face, or the distance between the eyes and the mouth versus the length of the face. Results from Experiment 1 on the eye to mouth distance versus facial length illustrate the general nature of the results. From this quadratic relation we can compute that the ideal ratio for the distance between the eyes and mouth versus facial length equals 0.36 as shown in Figure 5.

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