THE INTUITIVE LAW OF BUOYANCY

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Abstract

The quantitative relations that a person tacitly assumes to exist between the variables governing an ordinary physical phenomenon constitute the intuitive law of this phenomenon. Recent data suggest that intuitive laws match the respective physical laws when one can experience these quantitative relations. People typically have experience with only part of the quantitative relations between the variables that govern the phenomenon of buoyancy. The intuitive law of buoyancy was accordingly found to only partially match the respective physical law.

People tacitly assume that there are quantitative relations between the variables that govern an ordinary physical phenomenon. These tacitly assumed relations constitute what one could call the intuitive law of the physical phenomenon (Anderson, 1983b; Duit, Niedderer, & Schecker, 2007; Shanon, 1976; Smith & Casati, 1994; Viennot, 1979; Wilkening & Huber, 2002). We first exemplify three intuitive multiplicative laws that match the respective physical law.

The time a ball takes to roll down an inclined plane of fixed angle is

$$T = c \cdot \frac{D}{\sqrt{H}} \tag{1}$$

with c a constant, D the distance travelled by the ball, and H the initial height of the ball from the ground (Galileo's law). When T is the dependent variable and D and H are varied factorially, this law implies a fan of factorial straight lines. Anderson (1983a) had subjects rate the imagined time an object takes to reach the lower end of an inclined plane for fixed perceived values of D combined with fixed perceived values of H. The obtained factorial curves formed a fan of straight lines revealing an intuitive law for the imagined time of travel of a ball along an inclined plane formally identical to the respective physical law.

The minimum force necessary to slide a prism on a horizontal board is

$$F = c \cdot W \tag{2}$$

with c a constant and W the weight of the prism (Amontons' law). For a fixed roughness of prism, the curves relating F to W for each different roughness of board are straight lines fanning out from the origin of the diagram. For a fixed roughness of prism and with W and the roughness of board varied factorially, Corneli and Vicovaro (2007) had each subject rate the imagined friction of the prism on the board after the subject had lifted the prism and felt how rough the surface of the board was. The obtained factorial curves formed a fan of straight lines manifesting an intuitive law of friction formally identical to the respective physical law.

For a spring hanging from a fixed support, suspending a load of weight W from the lower end of the spring causes the length of the spring to increase from L to $L + E$ with

$$E = c \cdot L \cdot W \tag{3}$$

and c a constant of proportionality incorporating the effects on E of factors other than L and W (Hooke's law). The curves relating E to L for each W are straight lines forming a fan. With L and W varied factorially, Cocco and Masin (2010) had subjects lift a load and, simultaneously, look at a spring and rate the imagined elongation of the spring that would occur in case the load was suspended on the lower end of the spring. Factorial curves relating the ratings of imagined elongation to subjective weight for each fixed L formed a linear fan of straight lines revealing an intuitive law of elasticity formally identical to the respective physical law.

Interpretation of intuitive laws

When we judge, we often behave as if we were tacitly adding, multiplying, or averaging some quantitative information (Anderson, 1981, 1996, 2008). Since we use these operations of information integration in various types of judgments, intuitive laws of physical phenomena could merely be integration operations used in judgments on physical phenomena (Anderson, 1983a).

It could be that an intuitive law matches a physical law of the same mathematical form when this physical law becomes associated by learning with an integration operation of the same mathematical form. The learning of this association could require extensive direct experience with the quantitative relations between the variables of the physical law. This association based on direct experience may have occurred for the laws discussed previously: it is in fact a direct experience that increasing the slope of an inclined plane causes a proportionate reduction in the time of travel of an object down the plane (Galileo's law); increasing the weight of an object causes a proportionate increase in the push needed to slide the object on a surface (Amontons' law); and increasing the pull needed to stretch a spring causes a proportionate increase in the length of the spring (Hooke's law).

We explored the possibility that an information integration operation becomes associated with a physical law of the same mathematical form after subjects have had extensive experience with the quantitative relations between the variables of the law. We did this by determining the intuitive law of buoyancy considering that adults, although they had extensive experience with the behavior in water of objects of different weights and sizes, should not have had this extensive experience with objects in fluids other than water.

Archimedes' law

For an object of volume V and weight W freely immersed in a fluid, the volume V_i of the immersed part of the object displaces an amount of fluid of volume V_i and weight W_i. Buoyancy occurs when $W = W_i$. Multiplying W and W_i by V / V_i and rearranging, one obtains the law

$$P = \frac{W}{V \cdot D} \qquad (4)$$

with $P = V_i / V$ the proportion of volume of the immersed part of the object over the volume of the object and $D = W_i / V_i$ the density of the fluid.

The following experiment explored whether subjects tacitly knew the quantitative relations expressed in Equation 4.

Experiment 1

Subjects and stimuli

Ten psychology undergraduates served as subjects. Each stimulus consisted of the combination of two objects: one of twelve rectangular prisms covered with uniform black adhesive plastic and one of three bottles made of colorless transparent plastic. The dimensions of prisms were 3 × 4.2 × 10, 5 × 6.2 × 15, or 7 × 8.2 × 20 cm. For each of these volumes, the weight of the prism was 185, 240, 310, or 380 g. Each bottle had width and height of 5.5 and 15 cm, respectively. It was filled halfway with tea or shampoo or honey, each with about the same brownish yellow color. These fluids had measured viscosities of about 0.01, 20, and 100 P, respectively. The emission of smell from the bottles was prevented by closing the bottle caps tightly.

Procedure

On each trial, the experimenters presented first one bottle and then one prism to the subject as follows. A screen was hiding all the bottles from the subject's view. One experimenter took one bottle from behind this screen holding the top of the bottle with his hand. He extended this hand toward the subject and then tilted the bottle from vertical to about 45 degrees on the fontal parallel plane of the subject, once to the left and once to the right, so that the subject perceived the viscosity of the fluid. Stevens and Guirao (1964) showed that this kind of presentation adequately produces the perception of viscosity. The bottle was so presented for about 2 sec, after which the experimenter hid the bottle again.

A screen was hiding all the prisms from the subject's view. Immediately after the presentation of the bottle had ended, the other experimenter took one prism from behind this screen and handed it to the subject keeping the longer axis of the prism either vertical or horizontal. The subject was asked to hold the prism with the orientation shown by the experimenter. The subject was then asked to (i) imagine the prism placed on a large horizontal surface of a deep fluid equal to the fluid contained in the bottle that was presented immediately before the prism; (ii) silently evaluate whether or not the prism would float; and (iii) estimate the percent of volume of the imagined immersed part of the prism over the volume of the prism. The subject was informed that "0%" meant that the prism would be completely above and that "100%" meant that it would be completely under the imagined surface.

The 72 stimuli were presented to each subject in random order (2 orientations, 3 viscosities, 3 volumes, and 4 weights). Before the experiment, three stimuli selected at random were presented to the subject to familiarize the subject with the procedure. The duration of the experiment varied from 15 to 26 min.

After the experiment, the subjects were asked in this order: Do you remember your studies on (i) Archimedes' law and on (ii) buoyancy (yes, no, or little)? Is the volume of the immersed part of a floating prism increased, decreased, or unchanged when (iii) prism weight increases for fixed fluid density and prism volume, when (iv) prism volume increases for fixed fluid density and prism weight, or when (v) prism orientation changes from vertical to horizontal for fixed fluid density, prism weight, and prism volume?

Results

Figure 1 shows the results. The mean estimate of P—percent of volume of the part of prism imagined to be immersed in fluid over the perceived volume of prism—calculated across orientation of prism is plotted against the weight of prism for each volume of prism (left) and each viscosity (center) and against the volume of prism for each viscosity (right).

A 2 (orientation) × 3 (viscosity) × 3 (volume) × 4 (weight) analysis of variance showed that the main effect of orientation was not significant [$F(1,9) = 1.3$]. Only the orientation × volume and volume × weight interactions were significant [$F(2,18) = 3.6$ and $F(6,54) = 2.6$, $p < .05$, respectively]. Only the linear-linear component of the orientation × volume interaction and only the quadratic-linear component of the volume × weight interaction were significant [$F(1,9) = 6.6$ and $F(1,9) = 6.0$, $p < .05$, respectively].

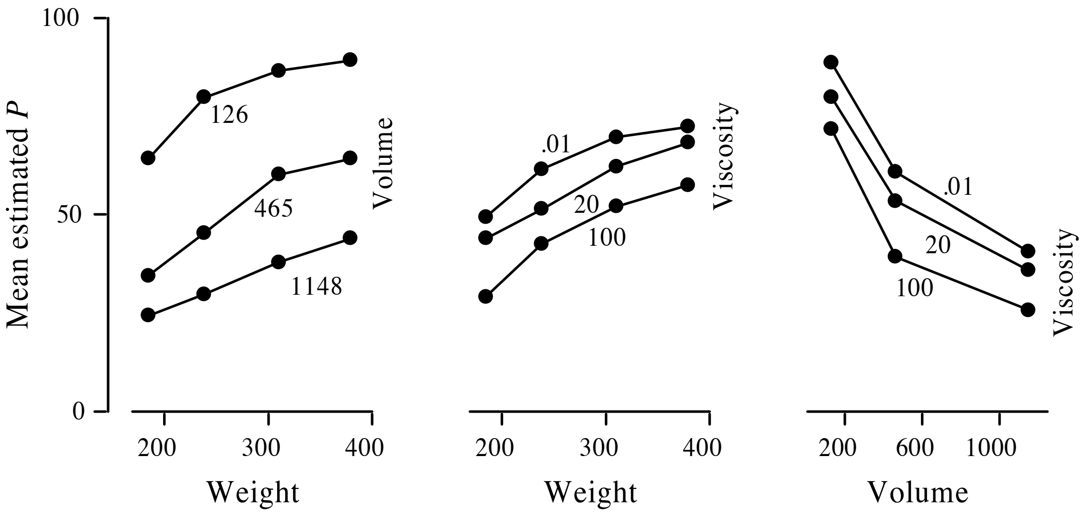

Fig. 1. Mean estimated proportion of volume of the imagined immersed part of a prism over the volume of the prism as a function of the weight or volume of the prism. The parameter is the volume of the prism or the viscosity of the fluid.

When the data for the smallest volume were excluded, the analysis of variance showed that only the volume × weight interaction [$F(3,27) = 6.33$, $p < .005$] and that only the linear-linear component of this interaction were significant [$F(1,9) = 18.13$, $p < .005$].

In Figure 2, mean estimated *P* calculated across weight and viscosity is plotted against the volume of the prism. For the smaller prism volumes, the results show that subjects tacitly believed that prisms were immersed more when they were vertical rather than horizontal. Corneli & Vicovaro (2007) similarly found that subjects tacitly believed that a prism on a horizontal board slid better when it moved along the longer axis of the contact surface.

The answers to the questions asked after the experiment were the following: (i) 4 subjects said that they remembered, 2 that they remembered little, and 4 that they did not remember about their studies on Archimedes' law; (ii) 3 subjects said that they remembered, 2 that they remembered little, and 5 that they did not remember about their studies on buoyancy; (iii) all subjects said the immersed prism volume increases as prism weight increases; (iv) 6 subjects said that the immersed prism volume decreases, 2 that it increases, and 2 that it remains invariant as prism volume increases; and (v) 8 subjects said that the immersed prism volume is larger and 1 said that it is smaller when the orientation of the prism is vertical, and the remaining subject said that prism orientation does not affect immersed prism volume.

Experiment 2

We repeated Experiment 1 with 10 different subjects, prisms presented only in vertical position, fluids with higher viscosities and light gray color, a large black vase to be imagined filled with fluid, and minor differences in the instructions. The obtained factorial graphs were practically the same as those in Figure 1. In agreement with the results of Experiment 1, the analysis of variance showed that the volume × weight interaction and its quadratic-linear component were significant when all volumes were considered and that only the volume × weight interaction and its linear-linear component were significant when the data for the smallest volume were excluded.

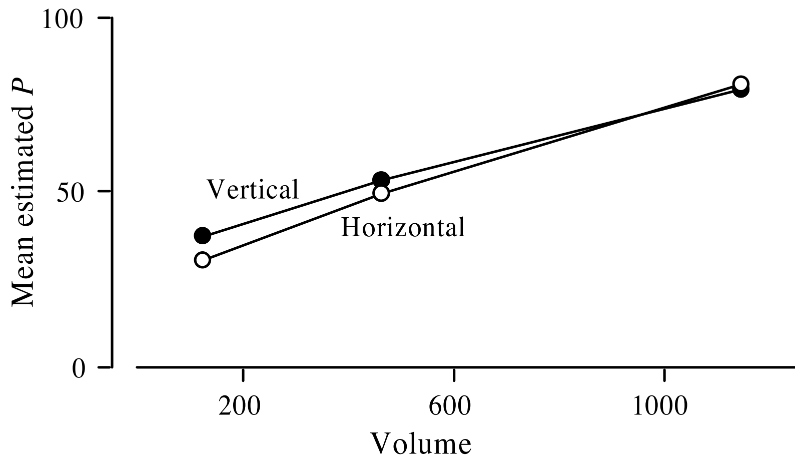

Fig. 2. Mean estimated proportion of volume of imagined immersed part of prism over volume of prism as a function of volume of prism when the prism was vertical or horizontal.

Discussion

In the left diagram in Figure 1 the graph for the smallest volume shows a ceiling effect. Due to the size-weight illusion, the perceived weight of smaller prisms was larger than subjects expected. This heavier perceived weight could have caused the ceiling effect, due to the obvious tendency of heavier prisms to be judged more often as totally immersed. When the data for the smallest volume were excluded, in both experiments the volume × weight interaction and its linear-linear component were significant with all the other interactions and respective components not significant. With all interactions involving viscosity and the respective components not significant, these results reveal the intuitive law of buoyancy

$$S_P = \frac{S_W}{S_V} + S_D \tag{5}$$

where S_P is the proportion of the imagined immersed volume over the perceived volume of the prism, S_W and S_V are the perceived weight and volume of the prism, respectively, and S_D is the perceived density (viscosity) of the fluid.

The primary finding of this study was that the intuitive law of buoyancy only partially matched Archimedes' law. That is, the quantitative relation tacitly assumed to exist between weight and volume agreed with physical law while the quantitative relation tacitly assumed to exist between these variables and density did not. Since subjects probably had no extensive experience with buoyancy in fluids other than water, they probably had learned to associate a ratio integration operation to the buoyancy phenomenon with respect only to the variables of weight and volume.

Why did subjects tacitly assume that the effect of density on buoyancy was additive even if they most probably never had any extensive experience with buoyancy in fluids other than water? We offer the following answer. There seems to be a hierarchy of preference for integration operations. For example, for the judgment of the area of rectangles, Anderson and Cuneo (1978), Cuneo (1980), and Wilkening (1979) have found that 10-year-old children and adults correctly use the multiplicative integration operation (height × width) while 5-year-old children use the additive integration operation (height + width). Since 5-year-old children probably had had little or no prior experience in judging the area of rectangles, they probably used the additive operation rather than another operation because the additive operation was the highest in the hierarchy of preferred operations. Accordingly, Wolf & Algom (1987) have found that 10-year-old children, who had already learned to use the multiplicative integration operation to judge the area of rectangles, went back to using the additive integration operation when they were faced with the more unusual problem of judging the volume of rectangular prisms (height + width + depth).

Thus, since the additive integration operation would be the highest in the hierarchy of preferences, this operation would be preferred over other integration operations when learning has not occurred or has occurred incompletely. The highest preference for the additive integration operation may explain why the subjects in the present study behaved as if they believed that the density of the fluid had an additive effect on buoyancy notwithstanding that they probably had no prior extensive experience with buoyancy in fluids other than water.

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