

COMPARING AND ESTIMATING EFFORTS

William M. Petrusic
Department of Psychology, Carleton University
bill_petrusic@carleton.ca

Abstract

The ratios and the differences of forces required to depress a lever were factorially varied in an effort discrimination task. Area under the ROC increased with increases in both the ratios and the differences. Accordingly, with the psychophysical function $\Psi(x)=Ax^\beta + C$; $\beta, C > 0$; either the Ratio model $D(x,y) = F[\Psi(x)/\Psi(y)]$ or the Power Difference Theory, $D(x,y)=G[\Psi(x) - \Psi(y)] = G[A(x^\beta - y^\beta)]$, with $\beta < 1$ is permitted. Alternatively, the difference model with logarithmic psychophysical function, $D(x,y)=G[\log(x + B) - \log(y + B)]$ with $B>0$, is also an admissible model. In a second experiment, magnitude estimates of efforts were obtained. The average exponent was 1.53. Consequently, the Power-Difference model is rejected. If, however, the exponent obtained from magnitude estimation arises as a consequence of matching subjective number with subjective intensity and with a subjective number exponent of, say, 0.5, then the Power-Difference model can be retained.

A number of long standing and currently popular models of the comparison process assert that some response measure, $R(x,y)$, on a pair of stimuli with physical magnitudes, x and y , is given by $R(x,y)=F(s(x),s(y))$, where F is a strictly monotonic function of the sensations, $s(x)$ and $s(y)$, independently evoked by stimuli, labeled x and y , respectively; i.e., the function F is said to be decomposable. Various models of comparative judgement, differing only in detail, either assume F is a function of the ratio $S(x)/S(y)$ (e.g., Luce, 1959) or of the difference $S(x) - S(y)$ (e.g., Thurstone, 1927) of the sensations associated with stimuli with physical magnitudes, x and y . Invariably, the psychophysical function, assumed to provide independent inputs to the decisional comparison process, is a logarithmic function or a power function.

Systematic variation of arithmetic relations defined on the physical magnitudes of the stimuli to be compared is quite revealing about both the process of comparison and the form of the psychophysical function. Table 1 summarizes the predictions arising when ratios, $r=x/y$, and differences, $d=x-y$, are systematically varied for the most common classes of models. The predictions are given for the case where the response measure is either $RT(x,y)$, the comparative response time, or $D(x,y)$, some index of discriminative sensitivity for the stimulus pair x and y .

Munsterberg (1894), Henmon (1906), and more recently, Petrusic and Jamieson (1979), demonstrate the exquisite sensitivity of comparative response times to arithmetic relations defined on supraliminally different visual extents. Each of these experiments clearly establish that response times monotonically increase as differences decrease with ratios constant and that response times increase as ratios approach one with differences fixed. As is evident from Table 1, these data severely restrict the classes of admissible models.

In fact, Petrusic and Jamieson (1979) show that since the exponent in the power function for visual extent was less than one for their data, only the Difference model with a power function (Power Difference theory) is permitted. Formally, *only* the case, $RT(x,y) = F(x^\beta - y^\beta) + C$, is permitted. The tests of the Power Difference theory by Petrusic and Jamieson were made with response times with supraliminally different

Table 1. Summary of Model Predictions

Choice model:	Psychophysical Function:	Prediction:
Ratio:	with r fixed, as $d \rightarrow 0$,	$D(x,y)$
$D(x,y) = F[\Psi(x)/\Psi(y)]$	$\Psi(x)=Ax^\beta; \beta>0$	is constant
	$\Psi(x)=A(x - \gamma)^\beta; \gamma, \beta>0$	increases
	$\Psi(x)=A(x^\beta - \gamma^\beta); \gamma, \beta>0$	increases
	$\Psi(x)=Ax^\beta + C; \beta, C > 0$	decreases *
Difference:	with r fixed, as $d \rightarrow 0$,	$D(x,y)$
$D(x,y)= G[\Psi(x) - \Psi(y)]$	$\Psi(x)=A\log(x+B) + C$	
	Case:	
	$B < 0$	increases
	$B = 0$	is constant
	$B > 0$	decreases,
	with d fixed, as $r \rightarrow 1$, $D(x,y)$	
	$\Psi(x)=Ax^\beta + C; \beta, C > 0$, with:	
	$\beta < 1$	decreases
	$\beta = 1$	is constant
	$\beta > 1$	increases **

* For the ratio models, $D(x,y)$ decreases as $r \rightarrow 1$ for all cases.

** For the difference models with logarithmic psychophysical function, $D(x,y)$ decreases with d fixed, as $r \rightarrow 1$ and with the power function with additive constant, $D(x,y)$ decreases with r fixed, as $d \rightarrow 0$.

visual extents. The present experiments provide tests of the Power Difference Theory using stimuli sufficiently close to one another that they are confused. Thus, the more typical classes of Generalized Fechnerian or Thurstonian models are tested here.

It is evident from Table 1 that when $\beta > 1$, the Power Difference Theory does not admit $D(x,y)$ to decrease as the ratio approaches one. If $\beta=1.0$, then $D(x,y)$ is constant whenever the physical difference is held constant. Consequently, the present experiments were designed to systematically examine variation in discriminative sensitivity upon variations in the differences and ratios of the magnitudes compared as defined by their physical measures in an effort discrimination task. Here, as distinguished from the classic experiments where the weights were hefted (e.g., Fechner, 1860), participants successively raised a counterweighted lever.

Method

Participants. Twenty four Carleton University students volunteered to serve for a two hour session in return for payment of \$5.00 and course credit. None had previously participated in a psychophysical experiment.

Apparatus. A lever and fulcrum device was used to produce the required variations in efforts. This device consisted of the following: a base, 61 cm in length, a round aluminum lever 75 cm long, a fulcrum to which the lever was attached by a round bolt at a height of 13 cm above the base and at a distance of 25 cm from the end of the lever, and two slider pieces 1.2 cm wide encircling the lever, one on each side of the fulcrum. Each slider piece was fitted below with a hanging attachment on which a weight was placed and fitted above with a thumbscrew which could be tightened to hold the piece in a fixed place on the lever. The lever was engraved with 1 mm divisions along the length; 1 cm divisions were also etched on the upper flat surface of the lever. The slider pieces had a 1 cm extension calibrated in 1 mm divisions. The scales on the lever and the scales on the sliders were designed for use in the Vernier manner to facilitate accurate positioning of the slider at specified points on the lever.

The slider on the shorter side of the lever was in a fixed position throughout the experimental session with a brass weight of 150 gm attached to the slider. The slider on the longer side of the lever had a 250 gm weight attached; the position of this slider was varied during the experimental session. The possible range of positions of this slider was restricted to 46 cm. The lever was restricted to movement of 2 cm upward and horizontal lateral movements were prevented by an upright stand with a slot for the lever at the last 1 cm of the lever.

Force was applied to the lever at its shorter end which was equipped with an electrical switch. This switch closed when pressure was applied to the hinged plate resting on the switch which was mounted on an expansion of the last 5 cm of the lever. A Data Gen Nova 1220 computer was used to control timing, event sequencing and the recording of responses and response times. The computer was interfaced to the lever, an Electrohome video monitor used to present the instructions on each trial, and a keyboard used for response buttons.

Stimuli. The pairs of forces to be discriminated on each trial were defined by varying the position of the slider along the lever such that the forces required to press the lever and to raise it just above the horizontal were also varied concomitantly. At the beginning of the experiment, the slider was set at the position closest to the fulcrum and the lever was balanced by moving the fixed position slider. Each force was then defined by moving the slider to a specified position on the lever.

Four sets of stimuli were used, providing tests at both the low and the high ends of the effort continuum. A fixed ratio of 5:4 or $r=1.25$ was used for two stimulus sets. Set 1, used at the low end, was defined by the following distances in cm from the balance position: [(4,5), (8,10), (12,15)], with differences of 1, 2 and 3 cm, respectively. Set 2, used at the high end, was defined by the following pairs: [(24,30), (28,35), (32,40)], with differences of 6, 7 and 8 cm, respectively. The ratios were varied with the difference held constant for the two remaining stimulus sets. Set 3, used at the low end, defined by the pairs: [(4,5), (6,7), (7,8)] used a constant difference of one cm and varied the ratio with ratios of $r=1.25$, 1.16 and 1.14. Set 4, used at the high end, uses these same ratios but maintains a larger difference of 7 cm and is thus defined by the pairs: [(20,25), (30,35), (40,45)].

Design. Twenty-four subjects were randomly assigned to the four experimental groups, defined by the above four sets of stimuli, with the constraint that each group consisted of three male and three female participants. The three stimulus pairs in each group were presented in two orders: on one-half of the trials, the stimulus requiring the smaller

effort was presented first and the stimulus requiring the larger effort was presented second. The form of the comparative was also varied: on one-half of the trials subjects selected the stimulus requiring the "greater" effort, while on the other trials, subjects selected the stimulus requiring the "smaller" effort. The factorial combination of pair by order by instruction yielded 12 different trial types. Each type was presented 5 times within a block of 60 randomly ordered trials. Each subject received four blocks of trials, two blocks in each of two sessions: replicating each of the 12 different trial types 20 times. The first session began with a set of 24 practice trials and were not included in the results.

Procedure. Each trial consisted of the following sequence: (i) the word READY appeared on the monitor for approximately 690 ms, then was removed; (ii) a 760 ms delay, during which the screen remained blank; (iii) the presentation of the instruction, SMALLER or GREATER, on the video monitor for ; (iv) a 745 ms delay; (v) presentation of the word PRESS, for 690 ms, directing the subject to press the lever; (vi) an interval of 1160 ms for the lever press; (vii) the word RELEASE for 690 ms; (viii) an interstimulus interval of 4144 ms; (ix) presentation of the word PRESS for the second stimulus.

Subjects indicated their response by pressing one of two response keys marked "first" and "second", respectively. In addition to this response, the subject was instructed to recall the instruction for that trial and to give a confidence rating on a five-point scale for the comparison response.

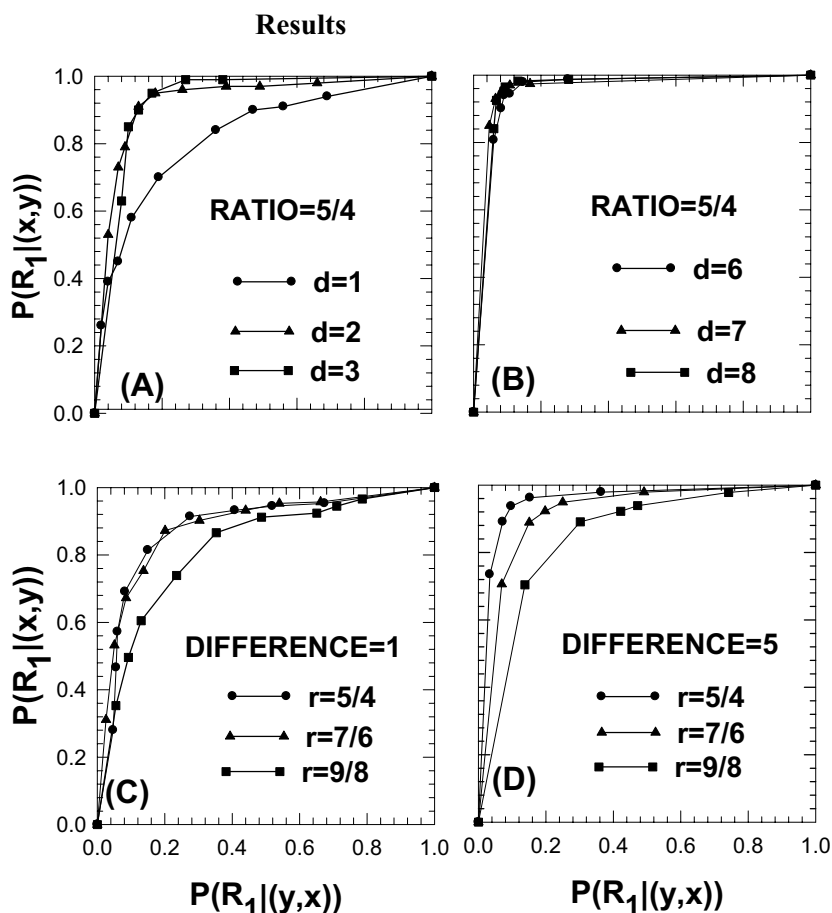


Figure 1. Confidence rating based ROC curves for each of the four groups.

As the plots in Figure 1 show, discriminative accuracy is marvelously sensitive to the factorial variation of the ratios and differences: accuracy improves as differences increase and as the ratios increase. Consequently, the variety of admissible models is greatly constrained. If the currently popular Power Difference theory is to be retained, then the exponent in the power function must be less than one.

Experiment 2: Magnitude Estimation of Efforts

In order to provide constraints on the empirical tenability of the PFSDT, the objective of this experiment was to obtain estimates of the exponent in the Power Function relation between the estimates of the effort required to depress the lever and the force required to raise the counterbalanced lever. Exponents were obtained for each participant; consequently, the frequency distribution of the obtained exponents could be examined.

Method

Participants. Twenty female and twenty male Introductory Psychology students worked for a single session of approximately 45 minutes in return for course credit.

Apparatus. The lever device used in Experiment 1 was employed

Stimuli and design. The forces to be estimated on each trial were defined by varying the position of the slider along the lever such that the force required to press the lever and to raise it just above the horizontal was also varied concomitantly. Each force was then defined by moving the slider to a specified position on the lever; forces are thus defined by the distance in cm of the slider from the balance position. The following distances of the slider from the balance position defined the forces for the magnitude estimation task: 2, 6, 10, 14, 18, 22, 26, 30, 34, 38, 42 cm. On each of six blocks of trials, each of the 11 forces was presented and the first block was viewed as practice. Participants were instructed to provide numerical estimates of the effort required to raise the lever and the shortest distance served as modulus with a value of 10.

Results

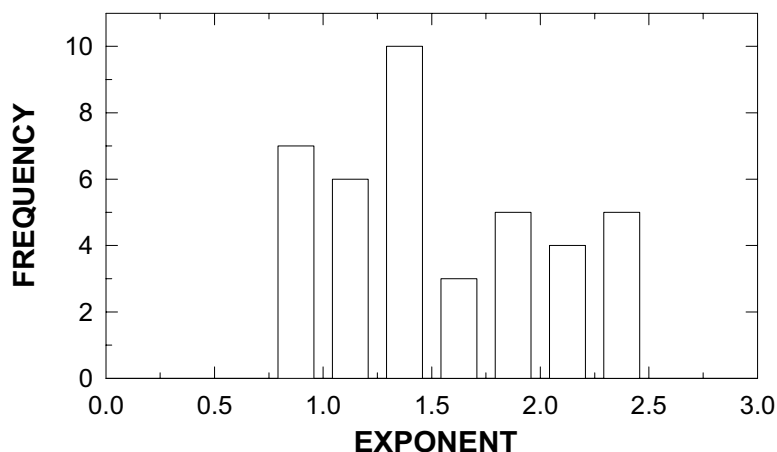


Figure 2. Frequency distribution of power function exponents. Mean=1.54, Median=1.41, Std. Dev=0.57, Min=0.77, Max=2.89, N=40.

If the psychophysical function for the subjective representation of number is a power function and magnitude estimation requires observers to match the subjective sensation S of stimulus with intensity I with subjective numerical magnitude, $S(N)$, then (see Krueger, 1989) $S(I) = aI^\beta = S(N) = cN^n$. Hence, $N = dI^{\beta/n}$, where $d = a/c$. If b is the observed exponent, then the “true” exponent, $\beta = nb$. Typically, n is near .50. Consequently, $\beta = .5b$. Here as is evident in Figure 2, the mean of the distribution of exponents is 1.54 and the median is 1.41. Thus, on the subjective sensation matching view of magnitude estimation, the “true” exponent, $\beta = .5(1.41) = .705$. On this view, the Power-Difference theory can be retained. On the other hand, if the magnitude estimates are taken at their face value and the subjective matching view is denied, then an exponent of 1.41 requires the rejection of the Power-Difference theory.

If the “true” exponent is less than one, then the present findings in support of Power Difference theory converge nicely with those obtained in the visual domain by Parker, Schneider, & Kanow (1974), Petrusic & Jamieson (1979), and Petrusic, Baranski, & Kennedy (1998) with perceived extents and Petrusic et al (1998) with remembered extents. In the auditory domain, Schneider (1980), and Schneider, Parker, & Stein (1974) show Power Difference theory to hold with comparisons of loudness intervals.

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