THE SPATIAL REPRESENTATION OF NON-SYMBOLIC NUMERICAL QUANTITIES

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Abstract.

Dehaene et al. (1993) have demonstrated an association between number magnitude and response position (SNARC effect). More recently, this association has been demonstrated also for non-symbolic quantities (see Shaki, Petrusic & Leth-Steensen, 2012). de Hevia & Spelke (2009) have used different numbers of dots as stimuli in a bisection task, showing a SNARC-like effect (but see Gebusi & Gevers, 2011). We investigated the association between representation of non-symbolic numerical quantities (dots) and response position in a simple detection experiment (see Fischer, 2003). The dots were used as prime, and the participants had to press a bar, as soon as they detected a grey square on the right or on the left. Our results (faster RTs for small quantities and target on the left, and for big quantities and target on the right) support the idea of a mental representation of non-symbolic quantities as a left-to-right oriented mental line.

Dehaene, Bossini and Giraux (1993) reported a spatial relation between number magnitude and its representation (i.e., Spatial Numerical Association of Response Codes: SNARC effect). In identification or comparison of numbers, participants were faster at processing large numbers (e.g., 9, presented in the center of a screen), when responses were executed in the right hemispace, whereas they were faster at judging smaller numbers (e.g., 1 presented in the center of a screen), when responses were executed in the left hemispace. The SNARC effect suggests that the representations of relatively small numbers magnitudes are spatially compatible with the left hemispace and those of relatively large numbers magnitudes are spatially compatible with the right hemispace (i.e., left-to-right oriented mental number line: MNL). This effect has been shown not only for number magnitude, but also for non-numerical ordered sequences. Gevers, Reynvoet, and Fias (2003) investigated the spatial organization of two non-numerical ordered sequences: names of the months, and letters of the alphabet. Gevers et al. asked participants to judge whether months presented in the centre of a screen came before or after "June", and to judge whether letters presented in the centre of a screen came before or after the letter "O". Results showed that the mental representation of these ordinal sequences could be spatially coded, because the names of the first months of the year were processed faster with responses executed in the left hemispace, whereas the reverse pattern was obtained for the last months of the year. Gevers et al. reported similar findings also on the task employing letters. In a similar vein, Rusconi et al. (2006) showed that even pitch is represented along a mental line. They explored the spatial representation of pitch height through the pairing of pitch to different response positions. In the first task, non-musicians were asked to compare the frequency of two pure tones. In the second task, non-musicians and musicians were asked to classify sounds as being produced by wind or percussion instruments. Results showed that the internal representation of pitch height was spatially organized, especially in participants with formal musical education (i.e., Spatial Musical Association of Response Codes: the SMARC effect). For a review of the SNARC effect, see Hubbard, Piazza, Pinel, and Dehaene (2005); for recent results on non-numerical magnitudes, and an up-to-date review, see Shaki, Petrusic, and Leth-Steensen (2012).

According to Walsh (2003), who has proposed a unifying framework called the ATOM (A Theory of Magnitude), the SNARC effect might be better understood as an instance of the SQUARC effect (Spatial Quantity Association of Response Codes), whereby any magnitude that is coded spatially or in action implies a relationship between magnitude and space. It follows that experiments in which responses are made to two or more magnitudes (independently of how they are coded), should show a magnitude priming on successive trials, regardless of the domain of coding. In this vein, de Hevia and Spelke (2009) have demonstrated, with a bisection task, that children show a SNARC-like effect with non symbolic (dots) representations of quantities, as well as for symbolic representations. However, Gebuis and Gevers (2011) have cast some doubt on those results, demonstrating that de Hevia and Spelke neglected a possibly confounding variable, that is, the extent of the area occupied by the dots (larger when the dots are more numerous).

On the basis of the studies by de Hevia and Spelke, and of Walsh's ATOM, we hypothesize that there is a spatial representation not only of the numbers, but also of the amount that they represent. We know from the literature that non-symbolic quantities are processed differently, depending on the number of items present. Amounts ranging up to 6-7 elements can be calculated very quickly, at a glance. This type of processing is called *subitizing*. On the other hand, amounts ranging from 9 elements onwards require a more accurate calculation; they cannot be calculated by means of a glance but must be counted. This process is called *counting*. In our study, we used the quantities that fall within the range of subitizing and of the counting to see how non symbolic quantities are spatially represented both in the range of subitizing and of counting.

EXPERIMENT

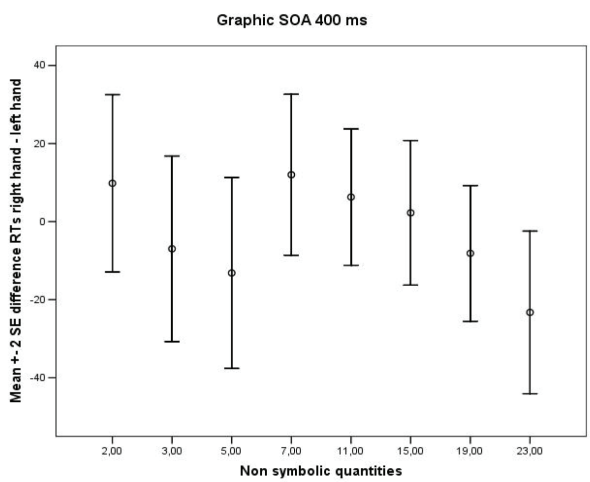
Method

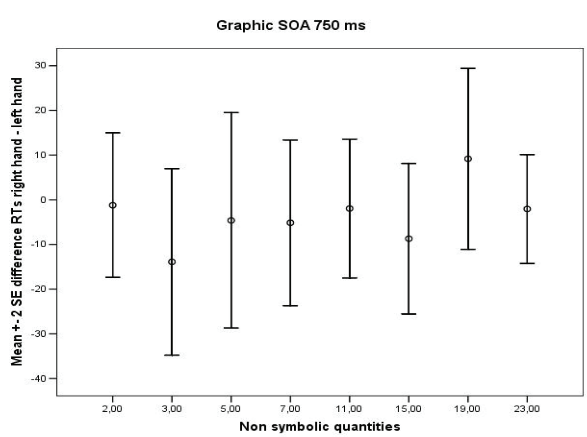
Sixteen students participated in the experiment, 1 male (age: 31 years) and 15 females (mean age: 21.5 years). Fourteen participants were right-handed, one was left-handed and only one was ambidextrous. All subjects had normal or corrected-to-normal vision. We used E-Prime software (Version 1.2., http://www.pstnet.com/eprime.cfm) to create and administer the experiment. Stimuli were displayed on a 26-inch (Quato Intelli Proof), with a 1024 x 768 pixel resolution (screen refresh rate: 75 Hz). The PC was a Dell Optiplex 382 Intel core (2) (RAM: 512 Mb), running Windows XP. We used a response box to record participants' responses. Each trial was comprised of three stimuli. The first stimulus was a fixation cross measuring 1° by 1°, that was placed between two boxes (dark grey). The second stimulus was an array of black dots (2, 3, 5, 7, 11, 15, 19, 23) that were presented in the middle of a grey circle. The third stimulus was a square target (light grey) that appeared in one of the two boxes. All stimuli were presented on a black background.

The experiment took place in a quiet, dimly lit room without environmental distractions. Participants sat in front of the monitor and were asked to put their index finger on the centre key of the response box. The viewing distance was 57 cm. Each trial started with a white fixation cross displayed for 300 ms at the centre of the screen, between two grey boxes, followed by an inter-stimulus interval (ISI) of 130 ms, consisting of a black screen. Then, the dots appeared for 200 ms. After an SOA, one of the two light grey target squares appeared for 1000 ms. Participants were asked to try to estimate the number of dots in their mind and then to press the centre key on the response box, when the target appeared. The inter-trial interval (ITI) was 1500 ms. The experiment was comprised of two sessions. In the first session, participants were asked to press the key with their right index finger when the square target appeared. In the second session, participants responded with the opposite hand. There was a short break between sessions, and the order of sessions was counterbalanced across participants. Each session comprised three blocks of trials (i.e., the training block and two experimental blocks). Each session started with the training block of eight trials. Thereafter, in each experimental block, the eight quantities were presented for ten times in random order (for a total of 160 trials).

Results

The data were analysed with regression for repeated measures (Fias et al., 1996; Lorch & Myers, 1990). The independent variable was the quantity values and the dependent variable was the difference between the median reaction time (RT) of the left target and the median RT of the right target: dRT = RT(right target) - RT(left target). In the first step, for each participant the median RT of the responses was computed for each quantity level, separately for left and right hand responses. Then, dRT was computed by subtracting the median RT of left target responses from the median RT of right target responses. In the second step, a regression equation was computed for each participant. In the third step, one-sample t-test were performed to test whether regression beta weights of the group deviated significantly from zero. The analysis revealed that the regression slopes (regression beta coefficients) were significantly different from zero, $t(15) = -2.697$, $p < .05$, only for the responses to the 400 ms SOA and for quantities within the range of counting. There was a left target advantage in processing the quantities 7-11 and a right target advantage in processing the quantities 19-23. These results can be interpreted as evidence that the non symbolic quantities in the range of counting were spatially represented on a mental line oriented from left to right (see the figure – top, SOA 400 msec, bottom SOA 750 msec).





Among other analyses, we have also performed an ANOVA for SOA at 400 ms and SOA at 700 ms, both for the quantities in the range of subitizing, and for the quantities in the range of counting. The results confirm those of the regression analyses, showing a significant interaction between target position and quantity, in the range of counting ($F(1, 15) = 7.272$, $p < .05$).

CONCLUSION

The SNARC effect shows that the numbers are mentally represented over the space, along a line oriented from left to the right (Dehaene et al., 1993). In the last few years, it has been demonstrated that this kind of representation holds not only for numbers, but also for other concepts (Rusconi et al. 2006, Shaki et al. 2012). Recently, it was asked if the same kind of spatial representation applies also to non-symbolic quantities, using a line bisection task. The evidence, however, is far from being unambiguous. The aim of our study was to determine whether the quantities given by dot patterns can be represented spatially in the range respectively of *subitizing* and *counting*. What we have found is that only in the range of counting are these quantities represented spatially according to a mental line that goes from left to right. Moreover, this effect is present only with a SOA of 400 ms, and not with a SOA of 750 ms. Probably this could be due to the fact that the SNARC effect, as shown by Fischer (2003), is one that becomes manifest after times of about 400-500 ms SOA. These results agree with the ones obtained by de Hevia and Spelke (2009), but do not answer to the objections raised by Gebuis and Gevers (2011), which require more research.

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EXTENDING SNARC: FROM SINGLE NUMBERS TO SEQUENCES

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Abstract

We found a Spatial Numerical Association of Response Codes effect for sequences of three single-digit numbers presented in English when Chinese speaking students determined if the sequence was in an ascending order.

People develop their understanding of numbers over time and experience. They begin with approximations and then learn to associate a single label with a specific quantity. A pre-school aged child playing with three erasers may know the word three corresponds to the three erasers, but may not yet understand that two comes before three, and four after. They eventually learn that numbers fit into a number line, both through recitation, and the visual reminder, often seen in elementary school class rooms. Siegler and Opfer (2003) showed that number ordering strategies change with age developing from either a log or a power function in grade 2 to a linear function after grade 6 through adulthood.

Dehaene, Bossini, and Giraux (1993), in a series of experiments, provided evidence for a mental representation of a number line. They found a magnitude of number by side of response interaction when deciding parity, this in spite of the fact that the magnitude of the number was irrelevant to the task. The time to respond was faster for small numbers when the side of response was on the left rather than the right. Conversely, the time to respond was faster for large numbers when the side of response was on the right rather than on the left. Coined the Spatial Numerical Association of Response Codes (SNARC) effect, it is typically reported as a difference in response times (dRT) between the left sided and right sided responses. They initially reported that magnitude, and not order, was the source of this new effect, as an experiment using letters as stimuli failed to elicit a SNARC effect.

The SNARC effect has been found to be affected by the direction of reading, the range of the number set used, and contrary to the original findings, has been extended to non-magnitude stimuli. Gevers, Reynvoet, and Fias (2003) found a SNARC effect for months of the year and letters of the alphabet (within order relevant and irrelevant tasks) and the days of the week (Gevers, Reynvoet, & Fias, 2004).

Previtali, de Hevia, and Girelli (2010) showed that a learned order of words could also elicit a SNARC-like effect. They interpreted this as the newly learned ordered sequences being learned in the same way as numbers and letters, and as such were represented in an ordered manner in spatial memory.

Prado, Van der Henst, and Noveck (2008) used relational reasoning to indirectly develop a sequential mental array of a seating plan. They found that the decisions about intermediate positions were noisy, that there was too much interference between the stimuli in the middle of the sequence. They therefore used only the end points in a subsequent analysis, and found a significant SNARC effect.

But not everyone accepts the mental number line as the under lying reason for the SNARC effect. Santens and Gevers (2008) argue for a three level model, wherein the intermediate level codes result in the effect.

If it is the case that we create an analog mental representation of learned sequences that maintains a culturally appropriate order, does this occur with groupings of numbers? In the following experiment sequences made up of three non-repeating numbers were presented. If the SNARC effect arises out of a correspondence between number position on a mental number line, then does it also extend to sequences of numbers when those sequences are made up of only small numbers or only large numbers?

Method

Participants

Thirty-four Carleton University students participated for partial course credit. Three student's data were removed from analysis due to non-performance of the tasks. The results from 19 women and 12 men with a mean age of 21.8 years (2.7 sd) were included in the analysis. Participant first-language breakdown: Arabic (6), Chinese (11), English (14). Three students were left-handed.

Equipment

A Pentium computer, a MultiLink LCD monitor, and a modified KeyTronic keyboard were used. Two keyboard keys were enabled as targets: A on the left side, and L on the right, with the other keys removed from the keyboard to avoid accidental key presses. The stimuli were controlled and the results collected using Superlab version 4.5 software. All instructions and stimuli were presented in Tahoma, regular, 20 point font, centered on the screen.

Stimuli

Stimuli triads were made up of three non-repeating single digit numbers from the number set {1,2,3,4,6,7,8,9}. Triads were categorised as in Table 1 These three numbers were spaced so as to be read as a single number made up of hundreds, decades and units. The practice also emphasised the separate and sequential nature of the numbers. These two sequences thus corresponded to the beginning and end of the single digit number set.. Category 1 composed of ordered and mixed triads from the subset of {1 2 3}. Category 2 were made up of {1 2 3 4} but not including Category 1. Category 3 included two numbers below 5, and one above. Category 4 included one number below 5 and two above. Category 6 was composed of the number set {7 8 9}. Category 5 was made up of numbers above 5 that did not include Category 6. The order of presentation of the tasks (Ascending, Descending) and side of response were counterbalanced between participants. Ascending had no descending sequences. Descending had no ascending sequences

Procedure

A centered fixation * * * was displayed for 1000 ms followed by a three-number sequence. If the stimulus triad was an ordered sequence the participant pressed either the right or the left button on the keyboard according to the instructions. The fixation then reappeared in the middle of the screen in preparation for the next trial. Three blocks of each condition were presented with a self-determined break. Six practice trials with feedback were done four times during the experiment, once before each new task. There were 72 stimuli x 3 blocks x 2 sides x 2 tasks = 864 responses. The experiment took approximately 45 minutes. Participants were told to be as fast and as accurate as possible.

Table 1. Examples of the three-number ordered and mixed stimuli. One and six are the end points of the Size factor. Size 1 & 2 contain non-repeating numbers under 5. Size 5 & 6 contain non-repeating numbers over 5. Size 3 & 4 contain non-repeating numbers crossing 5.

Type	1	2	3	4	5	6
Ordered	{1 2 3}	{1 3 4}	{3 4 7}	{3 7 8}	{6 7 9}	{7 8 9}
Mixed	{2 1 3}	{3 4 1}	{4 3 7}	{7 3 8}	{7 6 9}	{8 7 9}

Results

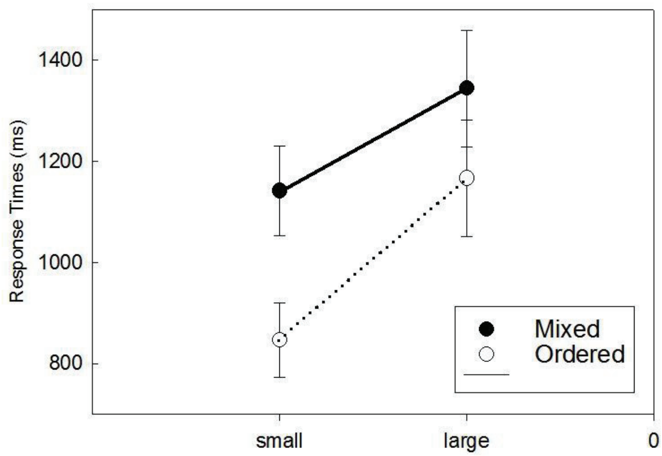

Figure 1. Interaction plot of Size (for the end points) and Ordered (ordered or mixed sequence). Errors bars are standard errors.

Incorrect responses were removed from the analysis (5.3% errors). Response times over the Mean RT plus 3 times the Standard Deviation for each participant were also dropped (1.9%) leaving 92.8% of the responses to be analysed.

A mixed model ANOVA was conducted with Language, Gender, and Handedness as between subjects variables. The main effects of Gender, Handedness, Language, Side of response, and Direction of Task were not significant. The main effect of Size was significant. Small triplets were responded to more quickly than large triplets, $F(1, 30) = 45.17$, $p < .00$. The main effect of Order was significant. Ordered triplets were responded to more quickly than mixed triplets, $F(1, 30) = 102.06$, $p < .001$.

The interaction of Size x Order was significant. As in Figure 1, ordered sequences were responded to more quickly than mixed sequences, and large three-number sequences took more time to respond to than small three-number sequences, $F(1, 30) = 8.34$, $p = .007$. The interaction of Size x Order x Direction trended towards significance, $F(1, 30) = 3.19$, $p = .084$.

SNARC

The Size x Side of Response x Direction (task) interaction was significant, $F(1, 30) = 4.34$, $p = .046$. The difference in response times (dRT) was calculated for each end point, using Right – Left hand response times for each participant (Figure 2). Individual regressions were run and tested for the Size by Side of response dRTs for Ascending, and for Descending following the process recommended by Lorch and Myers (1990). The single sample *t*-test for Ascending instruction slopes was not significant, $t(30) = -0.680$, $p = .501$. Whereas, for Descending it trended towards significance, $t(30) = -1.724$, $p = .095$.

Because the SNARC effect is dependent on the direction of reading and writing, a new set of single-sample *t*-tests was run taking writing direction into account. For participants who wrote left to right in their first language the regression slopes for dRT end points for Ascending instruction were still not significant, $t(24) = -0.650$, $p = .522$, 2-tailed. Descending regression slopes continued to trend towards significance, $t(24) = -1.967$, $p = .061$. For participants whose writing in their first language was right to left, Ascending, $t(5) = -0.222$, $p = .833$, 2-tailed, and Descending, $t(5) = -0.179$, $p = .865$, 2-tailed, regression slopes were not significant.

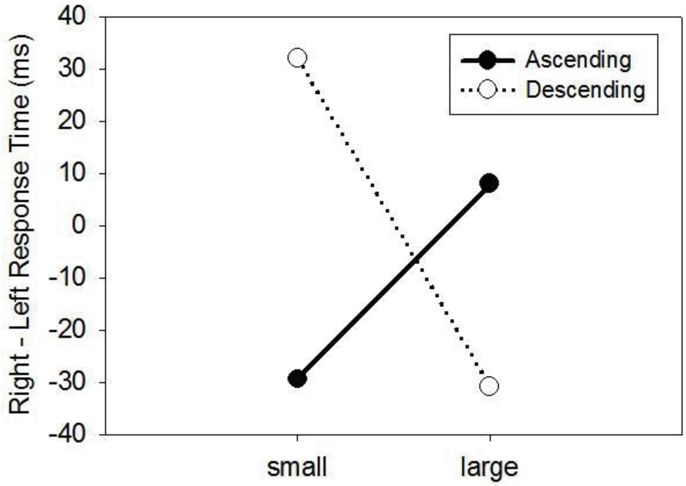

Figure 2. Interaction plot showing the end point dRTs for Ascending and Descending sequence instructions for all participants together

A further breakdown by language was conducted to determine if language groups had different patterns of responses to the stimuli. Table 2 shows the slope directions and significance testing results for each language group. The single-sample t-test was only significant for the Chinese language group for the Ascending instruction, $t(10) = -2.698$, $p = .022$.

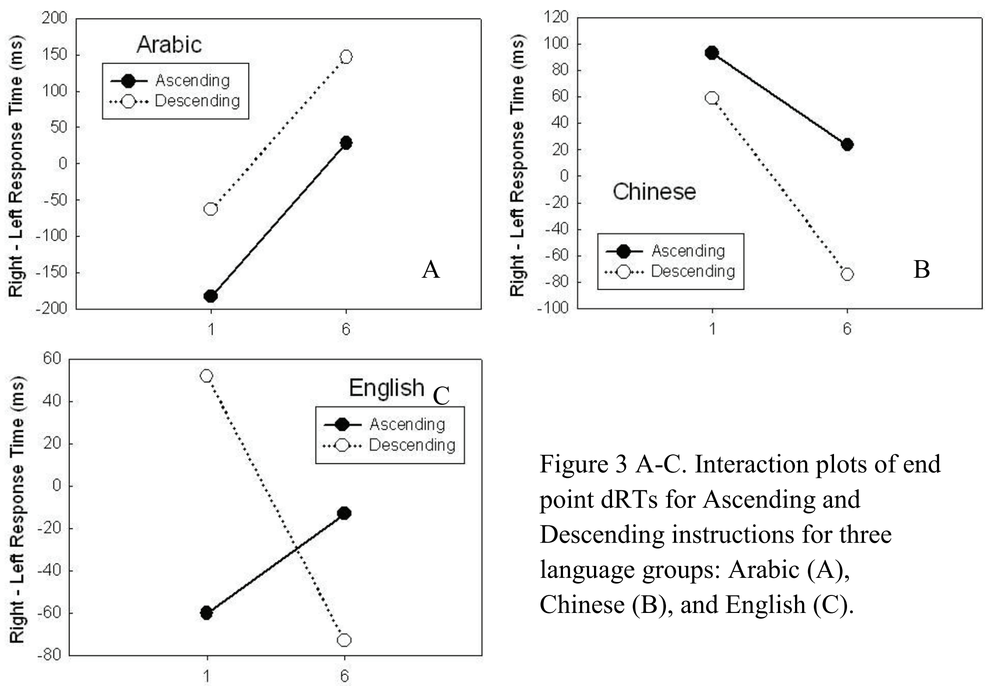

Figure 3 A-C. Interaction plots of end point dRTs for Ascending and Descending instructions for three language groups: Arabic (A), Chinese (B), and English (C).

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Table 2. Single-sample *t*-test (2-tailed) of individual participant regression slopes for both instructions by language for each directional task.

Language	*df*	*t* value	*p* value
Arabic			
Ascending	5	-0.222	.833
Descending	5	-0.179	.865
Chinese			
Ascending	10	-2.698	.022
Descending	10	-1.394	.194
English			
Ascending	13	1.675	.118
Descending	13	-1.336	.204

Discussion

The main focus of this experiment was to determine the explicit effect of order on the SNARC effect. Rather than have a single digit number, triplets were used (combinations of three single-digit numbers). A global SNARC effect was found at the end points of the possible sequences (the smallest or the largest numbers of the number set).

In the initial analysis, there appeared to be a crossover effect between the two tasks: deciding if the sequence was Ascending (Descending). Interestingly, the slopes were in the opposite directions to what would have been expected (Figure 2). In a regular SNARC effect, one might have expected that the left hand responses would have been faster for the small number sequence, and the right hand faster for the large number sequence within the Ascending task. Instead it appeared that for the all-participant analysis the Ascending task the dRT slope was positive, rather than negative. This pattern was reversed for the Descending task.

However, in the first two analyses, the full analysis, and the analysis based on reading direction, only the Descending task had a dRT slope that approached significance. This led to the third analysis broken down by first language of participants (Table 2). Only the dRT regression slopes for the Chinese students within the Ascending decision task passed the Lorch and Myers (1990) test.

Of interest are the different dRT slopes for the two tasks within the three languages as seen in Figure 3. For the dRT slopes for the English participants, the language of the experiment, there was a crossover effect. The English participants attended to the differences in the tasks changing their framework from which to make decisions. However, as in the overall analysis the expected slopes of the dRT are reversed, with Ascending being positive, and Descending negative.

One explanation for this apparent reversal might be understood through the process of reading. Reading tracks from left to right. With the reading of the third number a decision is made. The focus now is to the right, rather than the center of the screen. The decision is thus anchored on the right side, essentially flipping the framework. Rather than left to right, the decision is based on a right to left situation.

This explanation can also be applied to the Descending decision task. Since descending reverses the number line (9 8 7 6 …) the same process of reading left to right but deciding right to left flips the reversed number line, resulting in a dRT slope that is negative.

This interpretation works well for Figure 2 and the English component of Figure 3 where the dRT slopes for Ascending is positive and Descending negative. In the case of the Arabic students both Ascending and Descending slopes are positive. Reading direction in this language is right to left. Because only six Arabic students participated it is difficult to interpret what might be happening. The dRT slope for Descending is the reverse of that shown in English, as expected. But the dRT slope for Ascending is the same. It is possible that the students were using a mix of frameworks, with Ascending in English, but the more difficult instruction/task being recoded in Arabic.

The dRT slopes for both tasks for the Chinese students were negative. The dRT slope for Ascending was the reverse of the slope as in the English language analysis. Chinese students use the same left to right reading direction as English students. In this case the argument for task difficulty leading to a flipping of the framework cannot be made. It may be that Chinese students, who have learned their numbers through rote, treat Ascending and Descending in the same manner, without a reversal of the spatial framework.

The dRT slope for the Descending task was in the same direction for Chinese and English participants. This directional compatibility may be the reason why a SNARC effect was first noted for the Descending task.

Finally, only the analysis of slopes for the Ascending task for the Chinese students reached significance. Within this task, the Chinese students were faster at responding to whether a small number sequence was in order with their left hand, and faster with their right hand for a large number sequence.

Conclusion

A global SNARC effect was found for responses by Chinese students who were asked to determine if a sequence of three numbers were in ascending order.

Interestingly, patterns of dRTs varied by language. Unfortunately, this could not be further investigated due to the loss in power when the participant pool was subdivided into smaller language groups. Future research should look at increasing the number of participants within each language. It should also determine if difficulty of task has an impact when the task needs to be translated.

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