LOSS AVERSION AND THE LOCUS OF NONLINEARITY IN DECISION UNDER RISK: A TEST BETWEEN PROSPECT THEORY AND SP/A THEORY WITH FUNCTIONAL MEASUREMENT

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Abstract

Measuring loss aversion requires the capability to measure subjective values of loss and gain on a common unit scale with a common known zero. The present work rests upon a previously established ratio model for the integration of uncertain gains and losses, which allows that value and probability weighting functions be derived on a common metrics with a common zero. Observed functional shapes concurred generally with those predicted by Prospect Theory (PT) and disagree with a view of probability evaluation as the major, if not exclusive, source of nonlinearities in decision under risk, as advocated, for instance, in SP/A. Loss aversion was found for some but not for all subjects, which agrees with the role given to differential risk preferences in SP/A. For those subjects who exhibited loss aversion, its magnitude was consistently found inferior to that typically estimated with PT.

Loss aversion is a formal component of risk aversion in Prospect Theory (PT), both in its original (OPT: Kahneman & Tversky 1979) and cumulative (CPT: Tversky & Kahneman, 1992) forms, referring to the alleged higher subjective value of losses in comparison with commensurate gains. It is modeled in the value function of PT by a kink at the zero reference point, resulting in greater steepness for losses than for gains. Conceptually, it is not to be confused with the curvature of the basic utility function (assumed concave for gains and convex for losses), which is a distinct component of risk attitudes (the probability weighting functions being still a third one) (Köberlling & Wakker, 2005). This manifests in the use of distinct parameters for "loss aversion" and for "curvature" in the PT representation of the value function (Tversky & Kahneman, 1992):

$$v(\mathrm{x}) = \begin{cases} x^{\alpha} & \text{if x} \geq 0 \\ -\lambda\,(-x)^{\beta} & \text{if x} < 0 \end{cases} \quad [\alpha,\,\beta > 0;\, \lambda \text{ tipically} > 1], \qquad (1)$$

with v(x) denoting the subjective valuation of outcomes x, α e β the "intrinsic" curvature of the value function for gains and losses, respectively, and λ the coefficient of loss aversion. This difference can also be functionally appreciated in that loss aversion (λ) is thought of as a source of risk aversion for mixed (gain-loss) prospects, while convexity of the value function (indexed by β) constitutes a source of risk-seeking for pure loss prospects.

Despite their formal independence, measurement of loss aversion is empirically tied up to the measurement of utility/value across the relevant domain of gains and losses, as made clear by the definition of loss aversion set forth in Kaheneman & Tversky (1979):

$-v$(-x) $> v$(x), for all x > 0, or equivalently: v'(-x) $> v$'(x), if v has a derivative.   (2)

Legitimate comparisons of $v(x)$ with $-v(-x)$ obviously require that subjective values of gain and loss be measured on a common unit scale with a common known zero. Moreover, the condition "for all x > 0" requires that such measurement spans the domain of all relevant outcomes, which actually amounts to determining the entire curvature of the value function (a loss aversion coefficient could then be derived as the mean or median of the ratios $-v(-x)/v(x)$ across the full range of x). The noteworthy point is that measuring loss aversion (whatever the coefficient in use) and determining the functional shape of the value function share the same demanding measurement conditions; and that, through meeting them (a problem to which few solutions have been offered: see Abdellaoui et al., 2007), the assumptions concerning both loss aversion and utility curvature under PT can receive a simultaneous check.

This paper illustrates the estimation of subjective losses and gains on a common unit ratio scale, and the further derivation of subjective value and subjective probability functions, by means of functional measurement (FM) (Anderson, 1981; 1982). The approach rests on a relative ratio model for the integration of uncertain gains and losses previously established on a mixed regular game situation (Viegas et al, 2009), writing shortly as G/(G + L) (*G* = gains; *L*= losses), and more fully as:

$$R = \frac{PGxVG}{(PGxVG)+(PLxVL)},$$ (3)

with *R* = response, *PG* = probability of gain; *PL* = probability of loss; *VG* = gain value, *VL* = loss value. The algebraic ratio structure of the model allows the estimation of functional values of *G* and *L* on a common ratio scale, which can then be used, via the embedded multiplicative model of probability x value, to derive functional estimates of value and probability. These estimates presume nothing as to the parametric forms of the value function (or, the case being, of the probability weighting functions) and rest exclusively on the empirical validity of the compound ratio model for this task. As such, provided the model is valid, they enable proper testing of conjectured functional shapes.

Ability to adequately measure value provides of course more than a check of the "psychophysical" assumptions under PT. Comparative tests with influential theories which predict distinct functional shapes also become a possibility. SP/A (Security-Potential/Aspiration) theory is one such case (Lopes, 1996; Lopes & Oden, 1999). SP/A distinguishes itself from PT on fundamental respects, starting with the notion of "risk", which is of a motivational/attentional nature in SP/A and generally "psychophysical" (based on a principle of diminishing sensitivity) in PT. Importantly for present purposes, these differences reflect in predictions regarding the subjective functions of value and probability: (1) non-linear weighting of probabilities is assumed in both theories, and modeled similarly under a rank-dependent or (de)cumulative weighting rule; (2) non-linearity of decisional probabilities is distinctively used in SP/A as an alternative to using curvature in the basic utility function to model "risk attitudes"; (3) loss aversion, which determines a concave inflexion of the value curve about the reference point in PT, has no formal status in SP/A: while the aspiration level in SP/A corresponds properly to a reference point, it is not incorporated in the value function and it operates on a separate principle of stochastic control (maximizing the probability to achieve an outcome at or above the aspiration level). Points (2) and (3) together make possible to largely boil down the differences between these two frameworks, as far as functional shapes are concerned, to the prediction of a linear value function by SP/A, in contrast to the S-shaped value function predicted by PT. Checking these alternative predictions is taken in the following as a case study.

Method

Participants

30 naïve undergraduate students at the University of Coimbra (aged 18 to 25) were enrolled in the experiments in exchange for course credits.

Stimuli

Stimuli consisted of schematic representations of one roulette-spinner games. One disk divided along its vertical diameter in two sectors, the left one associated with losses and the right one with gains, was presented. Left and right sectors were coloured to different extents in red and green, respectively, causing the probabilities that a spinning arrow determined a loss (*PL*) or a gain (*PG*) to vary independently from each other, with a complementary probability (1- *PL* - *PL*) of a null outcome. Variable monetary upshots were associated with the loss and gain sectors – value of loss (*VL*) and value of gain (*VG*), respectively. The overall situation could be described as a mixed (gain-loss) regular (p + q < 1) two-outcome game.

Design and procedure

Two similar game situations were produced: (1) The "value condition", involving 2 probabilities (0.25, 0.85) and 5 values of gain and loss (+/- 15, 150, 500, 2000, 7000 €); (2) The "probability condition", involving 5 probabilities (0.05, 0.275, 0.5, 0.725, 0.95) and 2 values of gain and loss (+/-150, 2000 €). In both conditions, the factorial combination of Probability x Value gives rise to an overall 10(*expected gains*) x 10(*expected losses*) design, with a 2(*probability/*respectively *value*) x 5(*value/*respectively *probability*) subdesign embedded within each factor. This nested structure allows to alternatively describe the main design as a 2(*PG*) x 5(*VG*) x 2(*PL*) x 5(*VL*) in the "value condition" (respectively, 2(*VG*) x 5(*PG*) x 2(*VL)* x 5(*PL*) in the "probability condition") and enables a two-layered approach addressing either the molar or the embedded design. Participants performed on both conditions, with presentation order counterbalanced. Their task was to judge on a bipolar graphic scale the dissatisfaction-satisfaction each game would bring them if they were forced to play it. Games were never played, but merely judged "as if" they were going to be played.

Results

One first concern was with the adequacy of the integration model (Eq. 3 above) to the specific tasks (value and probability conditions). Both graphs in Fig. 1, corresponding to the major two-way factorial plots, display cigar-like patterns consistent with the relative ratio model. Multiplicative fan-like patterns (not presented here) were observed in the embedded probability x value designs in both conditions, which were supported by statistical analysis (significant bilinear components of the interactions, with null residuals over the other components). Plots of *VG* x *VL* in the value condition, and of *PG* x *PL* in the probability condition, exhibited the typical barrel patterns the model led to expect. Evaluating the fit of the model (which, being nonlinear, requires iterative procedures) was done as in Viegas et al. (2009), and proceeded on an individual basis. Mean RMSD values obtained for the fits were of 0.064 in the value condition and of 0.062 on the probably condition. Taking altogether, the basic integration model appeared as well warranted in both tasks.

As a characteristic feature of FM, parameter estimation is simultaneous with the empirical process of establishing the model's validity, rather than simply assuming it.
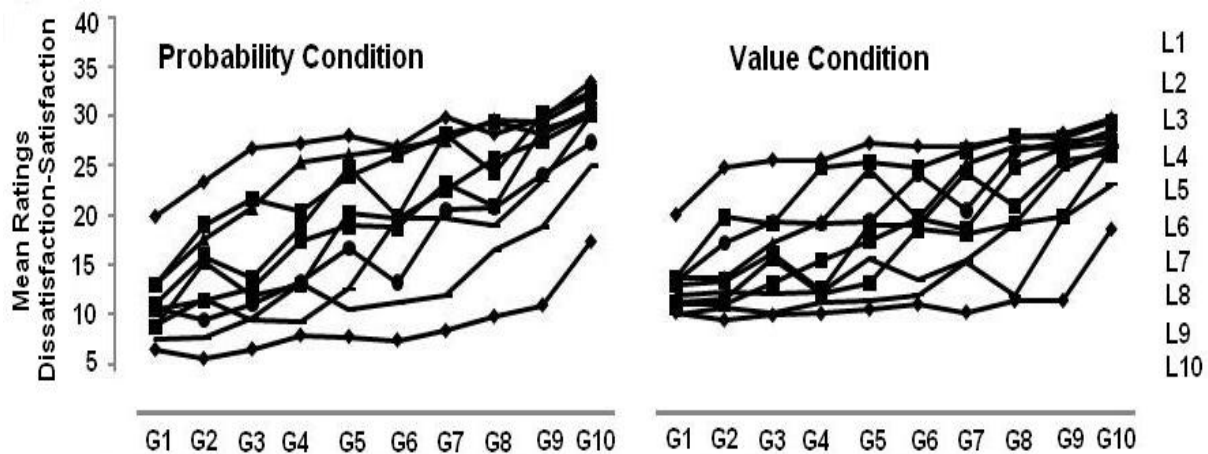
Figure 1. Factorial diagrams corresponding to the 10 (*Expected Gains:* G1 to G10) x 10 (*Expected Losses:* L1 to L10) overall design (increasing marginal means of *Gains* in the abscissa).

Estimates of G_i and L_i (the molar parameters) obtained in the process of fitting the model could thus be taken as legitimate, granted the validity of the relative ratio rule. Such estimates are on a ratio scale with an arbitrary unit (see Viegas et al., 2009, for details; also Anderson, 1982), thus affording meaningful comparisons between *G* and *L* on a common unit scale with a known zero. They were used as the basis for the subsequent derivation of functional measures of *probability* and of *value*, at the ratio level (no common unit across factors, though), resting now on the nested multiplicative rule (see. Viegas et al., 2009, for procedure; and Masin, 2004, on achieving ratio measures under the IIT multiplying model).

10 estimates of subjective value (5 for gains, 5 for losses) were derived this way in the value condition, and 10 estimates of subjective probability (5 for gains, 5 for losses) in the probability condition. Since a physical metrics of the stimuli was available (amount of money for "value", portion of coloured area for "probability") proper psychophysical functions of value and of probability could be plotted. These are presented in Figures 2 and 3, with adjusted trend lines and the corresponding parametric equations.
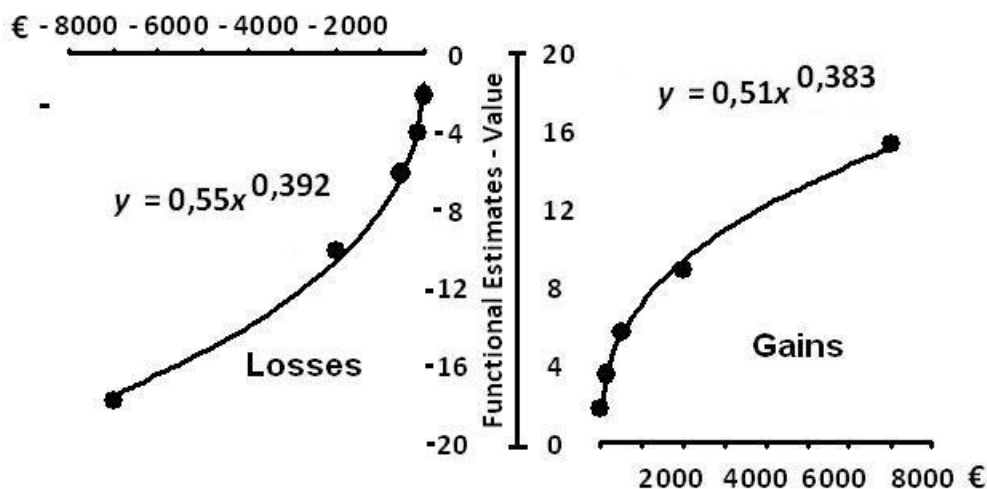

Figure 2. Psychophysical functions of *Value*. Functional estimates (aggregated) derived from the relative ratio model are pitted against monetary values. Points correspond to empirical data, lines to best fitting parametric trends.
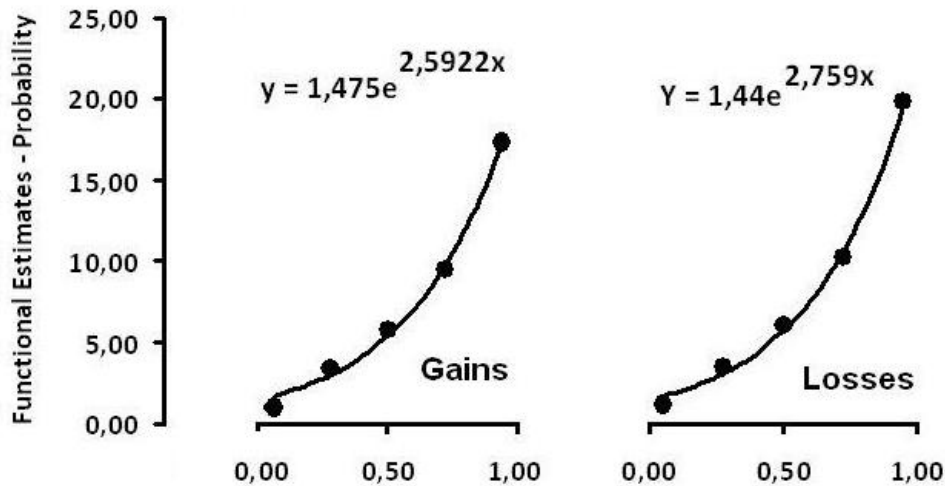
Figure 3. Psychophysical functions of *Probability*. Estimated functional values (aggregated) are on the ordinate and coloured proportion (in percentage) of the gain (loss) sectors on the abscissa. Points correspond to empirical data, lines to adjusted trends.

Functional Shapes

Figure 2 reveals pronounced non-linearity of the value curve, which appears concave for gains and convex for losses, thus S-shaped in all. Power functions provide the best fit to data. Both findings agree with PT and disagree with the assumption of linearity made in SP/A. A noteworthy point is the closeness of the power exponents estimated for gains and for losses. Since adjustments were performed on an individual basis, a paired-t test could be done which provided a null result (p = 0.596). This concurs with PT's assumption that the value function has a similar curvature for gains and for losses (even if the values found are – on the mean – considerably lower than the 0.88 typically taken as a reference).

Figure 3 illustrates an overall convex shape of the probability functions, consistent with the assumptions of OPT. Due to lack of a sensible way of mapping functional estimates on the range of 0 to 1, the underweighting/overweighting of probabilities cannot be addressed, and considerations must be limited to functional shapes. An exponential function does a good job at fitting the data. However, closer inspection shows the first data point to lie below, and the second and third data points to lie above the fitted line in both graphs (gains and losses), a suggestion of mild initial concavity which concurs with the inverse S-shape proposed in CPT (this tendency was also observed and widespread at the individual level)

Loss aversion

Loss aversion (LA) was computed as the mean of the ratio $v'(-x)/v'(x)$ (i.e., between the values of the derivative of the "utility" function for matched monetary losses and gains). The found mean was 1.18, a value much lower than the standard 2.25 assumed in PT, and not significantly different from 1, which would signal the absence of loss aversion (p = .07). The picture was however quite different at the individual level. Classifying subjects as "loss averse" (LA > 1) and "gain seeking" (LA < 1) resulted in mean coefficients of 1.38 and 0.85, respectively, for the two groups. Both coefficients were significantly different from 1 (p = 0.04, p = 0.02) as well as from each other (p = .023). Loss aversion was thus the case for a majority of subjects (≈ 64%) who focused more on losses than on gains, while ≈30% (gain seekers) did the exact opposite, and ≈ 6% accorded an even treatment to gains and losses.

Discussion

Measuring value/utility has proven a difficult problem under complex models such as PT, largely because of the assumed interference of other factors such as probability weighting and loss aversion. This work illustrates the possibility of desintegrating these different components using the framework of IIT and Functional Measurement, without imposing *a priori* constraints on the shape of either the value or the probability function. The results were consistent with the functional forms defended in PT, and disallowed the claim made in SP/A of a linear (or quasi-linear) value function. Similar curvature parameters were observed for gains and losses both in the value and the probability functions. This disagrees, in the later case, with PT, but similar results were found, for instance, by Abdellaoui et al. (2007) with a rather different methodology. As regards loss aversion, a non-negligible percentage of *gain-seeking* (the opposite of loss aversion) subjects was found, which is more in keeping with a dispositional account of loss aversion, as in SP/A, than with making it a structural feature of the value function, as in PT.

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