AN EXTENSION OF THE RELATIVE JUDGMENT MODEL

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Abstract

In the relative judgment model (RJM) of absolute identification, the difference between the current and previous stimulus is used, together with the feedback for the previous stimulus, to derive a response for the current stimulus. Thus the model is able to predict the large accuracy benefit when the previous and current stimuli are the same. However, a smaller but significant benefit is also seen when the stimulus two trials ago is the same as the current stimulus. Here, I show that although the original RJM cannot predict this effect, a modified version of the RJM can. In the modified RJM, participants sometimes retain the representation of the previous stimulus at the expense of storing the current stimulus. In this way, the stimulus two trials ago is sometimes used as the standard against which the current stimulus is compared. This modified model provided a significantly better fit for 72% of participants.

What is interesting about absolute identification? Miller (1956) observed that there seems to be a limit in our ability to identify stimuli that vary along a single psychological dimension. In an absolute identification task, stimuli vary along a single psychological continuum (e.g., tones varying in their pitch). Each stimulus is labeled by its rank order in the set. On each trial, a stimulus is selected at random from the set and presented. The participant must respond with their best guess of the label before the correct answer is given as feedback. Here is what is surprising. Even though the participant might be able to discriminate every adjacent pair of stimuli when they are presented one immediately after the other, they will have great difficulty in the absolute identification task if there are more than five or six items in the set. This limit holds for a wide variety of different dimensions across all five senses, suggesting that the limit is either duplicated in each sense or central in its locus (see Stewart, Brown, & Chater, 2005, for a review).

Absolute Judgment Models

Theoretical accounts of absolute identification may be divided into two types. In the first type, absolute identification judgments are made by comparing the stimulus with long-term memories of absolute stimulus magnitudes. In exemplar accounts (e.g., Kent & Lamberts, 2005; Nosofsky, 1997; Petrov & Anderson, 2005) a stimulus is compared to long-term memories of the absolute magnitudes of previous stimuli. In Thurstonian accounts (e.g., Braida & Durlach, 1969; Luce, Green, & Weber, 1976; Treisman, 1985), the sensory continuum is divided up into response categories by criteria that represent long-term absolute magnitude information. In limited capacity models (Marley & Cook, 1984), stimuli are compared to end anchors that provide long-term absolute magnitude information. In mapping models (e.g., Lacouture & Marley, 2004), stimulus magnitude is represented by the activation of a hidden unit, with long-term absolute magnitude information represented as the mapping between hidden unit activations and the output units.

The Relative Judgment Model

Stewart et al. (2005) presented a relative judgment model (RJM). In this model, absolute identification is achieved without reference to long-term magnitudes. Instead, following Laming (1997), participants are assumed only to be sensitive to the difference between consecutive stimuli. For example, consider the situation in which Stimulus 7 follows Stimulus 5. The sensation participants have is of a difference corresponding to two response categories. Because they know the previous stimulus was 5, and the current stimulus is 2 units higher, they respond 7. The model does not make any reference to absolute magnitudes. All that is required is some knowledge of the size of stimulus difference that corresponds to a single response unit.

The following mathematical description of the model is summarized from Stewart et al. (2005). Equation 1 describes the process of adding the the estimate of the difference between the current and previous stimuli $D^{C}_{n,n-1}$ on to the feedback F_{n-1} from the previous trial.

$$\boldsymbol{R_n} = F_{n-1} + \frac{D^{C}_{n,n-1}}{\lambda} + \rho \boldsymbol{Z} \tag{1}$$

λ represents the subjective size of the difference that corresponds to a single unit on the response scale. $\boldsymbol{Z}$ is a normally distributed random variable that represents the noise in the mapping process with a mean of 0 and a standard deviation of σ. The ρ term represents the range of responses available on any trial, given knowledge of F_{n-1} and the sign of $D^{C}_{n,n-1}$. Equation 2 describes how the difference between the current stimulus and previous stimulus is contaminated by the previous differences.

$$D^{C}_{n,n-1} = \sum_{i=0}^{n-2} \alpha_i D_{n-i,n-i-1} \tag{2}$$

The α coefficients are constrained to be in the range $0 \le \alpha \le 1$. The coefficient for the current difference α_0 is fixed at 1, and the remaining α coefficients are constrained to decay monotonically with increasing lag. The stimulus differences themselves are given by

$$D_{n,n-1} = A \ln\left(\frac{X_n}{X_{n-1}}\right) \tag{3}$$

where X_n is the physical magnitude of the stimulus on trial n (S_n) and A is a constant that depends on the sensory dimension. The random variable $\boldsymbol{R_n}$ from Equation 1 is partitioned into response categories by N - 1 criteria, labeled $x_1, x_2, ..., x_{N-1}$, that partition the response scale such that accuracy is maximized. The probability of a given response r is given by the total density of $\boldsymbol{R_n}$ within the range

$$x_{r-1} < \boldsymbol{R_n} < x_r \tag{4}$$

with the lower and upper bounds replaced by $-\infty$ and $+\infty$ for the lowest and highest responses respectively.

Because the RJM is a relative judgment model, it can account for the ubiquitous sequential effects seen in absolute identification. These effects account for a large amount of

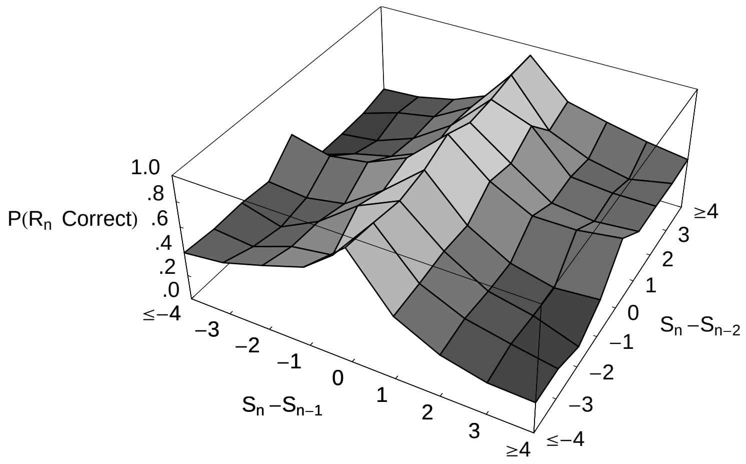

Fig. 1. Accuracy of R_n as a function of S_n - S_{n-1} and S_n - S_{n-2}.

the variability in responding on each trial. Thus, modeling sequential effects is likely to help understanding of why absolute identification performance is so poor when discrimination performance is so good. Typically, responses to the current stimulus are biased towards the preceding stimulus and away from those before that (e.g., Ward & Lockhead, 1970, 1971). Responses are also much more accurate when the previous and current stimuli are similar (e.g., Siegel, 1972; but see Stewart et al. for a discussion of some exceptions). The RJM accounts for sequential effects because it assumes that the perception of the difference between the current and immediately preceding stimulus is confused previous differences.

More Detailed Analysis of Sequential Effects

This paper is concerned with developing the account of sequential effects further. The data were taken from Stewart et al. (2005) Experiment 1. In this experiment, there were two different stimulus spacings and three different set sizes (6, 8, and 10 stimuli). Here, data from the Set Size 10 condition are presented, collapsed across the two different stimulus spacing conditions. Very similar patterns are obtained in the Set Size 6 and 8 conditions.

In Figure 1, the simultaneous effects of S_{n-1} and S_{n-2} are examined. The accuracy of the current response R_n is plotted as a function of the difference between S_n and S_{n-1} and the difference between S_n and S_{n-2}. The most prominent observation is a large increase in accuracy whenever $S_n = S_{n-1}$. There is also a much smaller accuracy advantage whenever $S_n = S_{n-2}$. (In Figure 1, trials when $|S_n - S_{n-1}| \geqslant 4$ and $|S_n - S_{n-2}| \geqslant 4$ are collapsed together because there are only a relatively small number of trials in this category.)

A (S_n - S_{n-1}) x (S_n - S_{n-2}) two-way ANOVA revealed a main effect of S_n - S_{n-1} [$F(8, 48) = 28.86$, $MSE = 2.65$, $p < .0001$], a main effect of S_n - S_{n-2} [$F(8, 48) = 5.31$, $MSE = 0.33$, $p < .0001$, and a significant interaction between the two $F(64, 384) = 2.42$, $MSE = 0.12$, $p < .0001$. The accuracy increase when $S_{n-2} = S_n$ was bigger when S_{n-1} was most different from both. The interaction also reflects the fact that when S_n - S_{n-1} and S_n - S_{n-2} have the same sign, accuracy is slightly higher than when they have the opposite signs because S_n is much more likely to be an edge stimulus in this situation, and accuracy is higher for edge stimuli.

The pattern of results in Figure 1 contrasts with that found in by Stewart and Brown (2004) in a binary categorization of 10 stimuli varying along a single dimension, with the lower 5 belonging in one category and the upper 5 belonging in a second category. In this experiment, under a relative judgment strategy, comparisons with particular stimuli can determine the category of the current stimulus. For example, if feedback indicates that the previous stimulus belongs in the lower category and the difference between the current stimulus and the previous stimulus is negative (i.e., the current stimulus is even lower) then the current stimulus *must* also belong in the lower category. Stewart and Brown found that,

Table 1. Median Best-Fitting Parameter Values, Averaged Across Participants

Parameter	Original RJM	Retention RJM
α_1	0.074	0.058
α_2	0.050	0.027
α_3	0.022	0.010
c	0.060	0.033
σ	0.236	0.221
λ	0.996	1.036
p_R	-	0.184

when S_{n-1} could be used in this way S_{n-2} had almost no effect. More surprisingly, when S_{n-2} could be used in this way S_{n-1} had almost no effect, suggesting that participants were basing their categorization of S_n entirely upon its relation to S_{n-2}. In these absolute identification data, even when S_{n-2} matches S_n exactly, S_{n-1} still has a big effect. For some reason, participants cannot or do not use S_{n-2} to categorize S_n nearly as much as one would expect given Stewart and Brown's data.

Fits of the Original RJM

The original RJM was fit to the data individually for each participant. For each participant and for each trial in the experiment, the RJM was used to predict the probability of the response given. The model's free parameters were adjusted to maximize the likelihood of each participant's data given the model. Because of model fitting time constraints, only the effect of stimuli up to four trials ago was considered, and earlier stimuli were assumed to have no effect. (S_{n-2} and earlier are assumed to affect R_n only in as much as the differences between these stimuli are assumed to contaminate the estimation of the difference between S_n and S_{n-1}, upon which responding is based.) Six parameters were allowed to vary α_1, α_2, α_3, c, σ, and λ. The median across participants of the best fitting values is given in Table 1.

Figure 2 shows the predictions of the best-fitting original RJM for each participant averaged across participants. As in the data, the original RJM predicts a large accuracy advantage when $S_n = S_{n-1}$. However, the original RJM fails to predict increased accuracy when $S_n = S_{n-2}$.

Fits of the Modified RJM

A modification of the RJM that allows it to predict an accuracy advantage when S_{n-2} and S_n are equal is reasonably straight forward. I take the advantage to indicate that, on some trials, S_{n-2} is used as the comparison item, and the difference between S_n and S_{n-2} is added onto the feedback F_{n-2}. One of the core principles of the RJM is that long-term absolute magnitude representations are not used. Thus I have chosen not to extend the model by allowing memory of many previous stimuli (presumably high quality for S_{n-1}, lower quality for S_{n-2}, and so on). As described in Stewart et al. (2005), there must be some representation of the absolute magnitude of S_{n-1} over the silent inter-trial interval so, in modifying the RJM, I assume that on each trial, the representation of the previous stimulus can be preserved in the store at the expense of storing the current stimulus.

After the response is given on each trial, the stimulus in the store is retained with probability p_R. Equivalently, the current stimulus replaces the stored stimulus with probability

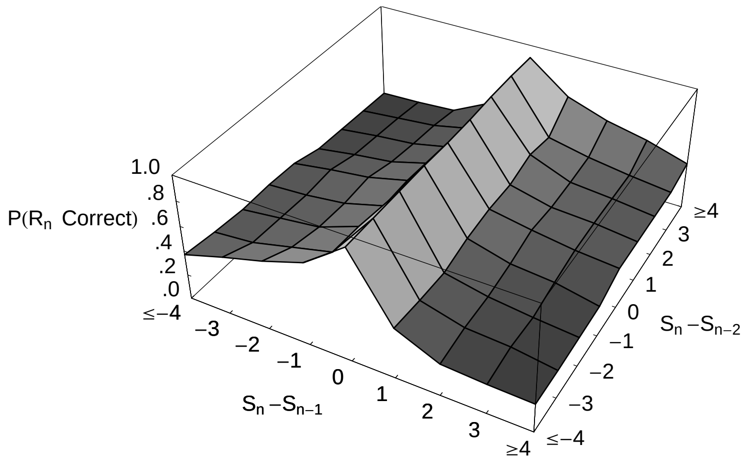

Fig. 2. The original RJM's predicted accuracy of R_n as a function of S_n - S_{n-1} and S_n - S_{n-2}.

1 - p_R. Equation 1 is modified so that F_{n-1} is replaced by $F_{n-L(0)}$, where the subscript $L(0)$ denotes the lag of the stimulus currently in the store.

$$\boldsymbol{R_n} = F_{n-L(0)} + \frac{D^C_{n,n-L(0)}}{\lambda} + \rho \boldsymbol{Z} \qquad (5)$$

Similarly, $D^C_{n,n-1}$ is replaced by

$$D^C_{n,n-L(0)} = \sum_{i=0}^{n-2} \alpha_i D_{n-i,n-L(i)} \qquad (6)$$

where the subscript $L(i)$ gives the lag of the stimulus in the store at lag i.

This modified model contains the original RJM as a special case when $p_R = 0$, and so nested model comparisons can be used to see whether allowing p_R to vary freely provides a significantly better fit to the data. The best-fitting parameters for the modified RJM are given in Table 1. The modified RJM provided a significantly better fit to 86 of the 120 participants' data sets from Stewart et al. (2005) Experiment 1. Figure 3 shows that the modified RJM can predict the increase in accuracy when $S_{n-2} = S_n$. In conclusion, by assuming that, on some trials, participants retain the p revious stimulus in a single-item-capacity short term store rather than admitting the current stimulus, the RJM has been extended to account for increased accuracy when the stimulus two trials back matches the current stimulus.

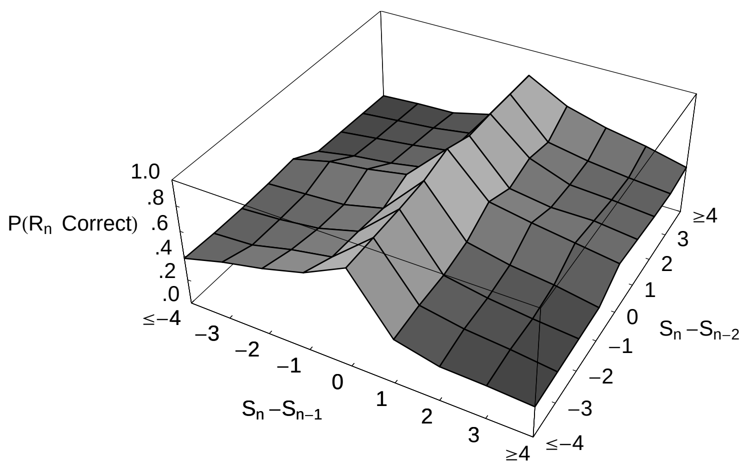

Fig. 3. The modified RJM's predicted accuracy of R_n as a function of S_n - S_{n-1} and S_n - S_{n-2}.

Acknowledgments

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